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Optimization

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The program for today

1. Optimization in a power systems context
2. Basic concepts & definitions in optimization
3. Examine some classical power system optimization problems
The temporal hierarchy of decisions

1. **Capacity Planning**
   Investment: How many? Size? Type? Other resources?

2. **Production Cost**
   How much will this system cost to operate?

3. **Hydrothermal Coord. Maintenance**
   When to use water? Shut down?

4. **Unit Commitment**
   Which resources to use given constraints (startup, ramp, etc.)

5. **Economic Dispatch**
   What output level for available resources?

6. **Load Flow**
   Where on the network does electricity flow? Losses?

7. **Protection/Stability**
   How to protect grid from damaging faults?

Source: Bryan Palmintier
The Short Blanket Theorem

Uncertainty

Network

Stochastic

Deterministic

Time Domain

Real-time (Economic Dispatch & Reserves)

Short-term (Unit Commitment)

Medium-term (Hydro-thermal, Maintenance Planning)

Long-term (Capacity Planning)

Single Node

Transport Model

DC Load Flow

AC Load Flow
All-purpose conceptual optimization model for planning & operation in the power sector
Resources “clear” the market in order of lowest to highest cost.

* Prices for renewable sources can be low or high.
Minimize:
\[ \sum \text{Operation costs} + \text{Investment costs} \]

Subject to:
- Supply = demand
- Operational constraint
- Reliability constraints
- Environmental constraints
Hard constraints vs cost penalties

Reliability constraints:
- Reserve margin/requirements
- Dynamic reserves

Environmental constraints/costs
- SO2/NOx
- CO2

Operational constraints
- Min/max power
- Max ramp up/down
- Minimum up/down times
Minimize:
\[ \sum \text{Operation costs} + \text{Investment costs} \]
Subject to:
- Supply = demand
- Operational constraints
- Reliability constraints
- Environmental constraints

Minimize:
\[ \sum \text{Operation costs} + \text{Investment costs} + \text{Costs associated with non served energy} + \text{Costs associated with environmental impacts} \]
Subject to:
- Supply = demand + non-served energy
- Operational constraints
The standard optimization problem

\[
\min_{x} c(x) \quad s.t.
\]

\[
g(x) = 0
\]

\[
h(x) \leq 0
\]
The standard optimization problem

If $X = (X_A, X_B)$, how would you represent this optimization problem graphically?
Formal definition of Optimization

• Goal
  – Find the value of the variables, so that the objective function is optimized, satisfying the constraints

• Objective function
  – quantitative measure of the system to be optimized (maximized or minimized)

• Variables:
  – These are the decisions to be taken, which affect the value of the objective function

• Constraints
  – relationships that the variables have to satisfy that render unfeasible some regions of the search space
The standard optimization problem

The *standard form* of a (continuous) optimization problem is

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad g_i(x) \leq 0, \quad i = 1, \ldots, m \\
& \quad h_i(x) = 0, \quad i = 1, \ldots, p
\end{align*}
\]

where

- \( f(x) : \mathbb{R}^n \to \mathbb{R} \) is the **objective function** to be minimized over the variable \( x \),
- \( g_i(x) \leq 0 \) are called **inequality constraints**, and
- \( h_i(x) = 0 \) are called **equality constraints**.

By convention, the standard form defines a **minimization problem**. A maximization
Optimization methods (1 of 3)

• **Linear Programming LP**
  - Continuous variables
  - Linear constraints

• **Quadratic Programming QP**
  - Linear/Quadratic objective function
  - Very powerful algorithms and commercial tools for solving LP problems
Linear Programming LP

• Mathematical formulation

  o **Variables**
    
    X, Y
  
  o **Objective function**
    
    Min a X + b Y
  
  o **Constraints:**
    
    X > 0, Y > 0
    X <= Xmax, Y <= Ymax
    c X + d Y <= e
    f X + g Y <= h
minimize\(_{x_1,x_2}\) \( z = -3x_1 - 5x_2 \)
subject to \( 3x_1 + 2x_2 \leq 18 \)
\( x_1 \leq 4, \quad x_2 \leq 6, \quad x_1 \geq 0, \quad x_2 \geq 0. \)
The concept of dual variable / shadow price
Economic dispatch
Minimize:
\[ \Sigma (\text{Var\_Cost}_g + \text{Fuel\_Cost}_g) \times \text{MW}_{g,h} \]
\[ + \text{VOLL} \times \text{NSE}_h \]

Subject to:
- Supply = demand + non-served energy
- Operational constraints:
  - Ramp rates
  - Min/Max power?
- Reliability constraints?
  - Reserves
- Environmental constraints?
  - CO\textsubscript{2}, SO\textsubscript{x}/NO\textsubscript{x}
ARGGG
Discrete variables!
Unit commitment
Minimize:

\[ \sum (\text{Var}\_\text{Cost}_g + \text{Fuel}\_\text{Cost}_g) \times MW_{g,h} + \text{Start}\_\text{Cost}_g \times \text{Startup}_{g,h} + \text{VOLL} \times \text{NSE}_h \]

Subject to:

- Supply = demand + non-served energy
- Operational constraints:
  - Ramp rates
  - Min/Max power
  - Minimum up and down times
- Reliability constraints?
  - Reserves
- Environmental constraints?
  - CO$_2$, SO$_x$/NO$_x$
\[4x + 2y = 32\]

\[3x + 2y = 27\]

\[x + y = 11.5\]

\((5, 6)\)
Optimization methods (2 of 3)

– Mixed Integer Programming – MIP

- Continuous variables
- **Integer/Binary variables**
- Linear constraints
- Linear objective function
- Limitations in the size of the problems to be solved, comparing to LP
ARGGGG
Non linearities!
Because of generation...
Non-convexities: heat rates

<table>
<thead>
<tr>
<th>NGCC</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat rate</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Minimum load

Efficiency

Btu/MWh

MW
Generator Costs

\[ C_i(P_{Gi}) = C_{0i} + a_i P_{Gi} + \frac{1}{2} b_i P_{Gi}^2 \]
Non-convexities: heat rates

**FIG. 2.9** A combined cycle plant with four gas turbines and a steam turbine generator

**FIG. 2.10** Combined cycle plant heat rate characteristic.
... and because of the network
Where:

• \( G_{ij} \) & \( B_{ij} \): elements of the admittance between the buses \( i \) & \( j \)
• \( \theta_{ij} \): difference between the voltage angles \( \theta_i \) & \( \theta_j \) at the buses \( i \) & \( j \)

Solving the load flow consists of finding the set of phase angles \( \theta_i, i=1, 2, \ldots, n_L+n_G \), and the set of voltage magnitudes \( V_i, i=1, 2, \ldots, n_L \), satisfying this system of \( 2n_L+n_G \) equations.
DC Load Flow

• A reasonable linear approximation of AC load flow:
  • *Ignores* voltage differences and reactive power flows at all nodes. (Assumes $V_i = 1$ at all buses)

Load flow equation becomes:

$$P_{ij} = G_{ij}(\cos \theta_{ij} - 1) + B_{ij} \sin \theta_{ij}$$
DC Load Flow

• A couple more simplifications:
  • \( \cos \theta_{ij} \approx 1 \) & \( \sin \theta_{ij} \approx \theta_i - \theta_j \)
  • \( B_{ij} = x_{ij}/(r_{ij}^2 + x_{ij}^2) \approx 1/x_{ij} \)

where: \( X_{ij} \) is the reactance of the branch from node \( i \) to \( j \)

DC load flow equation then simplifies to following \textbf{linear} expression:

\[
P_{ij} = (\theta_i - \theta_j)/x_{ij}
\]
Even Simpler Approximations…

• A simple **transport model**:
  • Only takes advantage of Kirchoff’s 1\textsuperscript{st} Law: the sum of power flows entering each network node must equal the sum of power flows exiting the same node.
  • Simple constraints specify maximum flows for each individual line.
  • Losses can be approximated from active power flows.
Optimal Power Flow (OPF)
Minimize:

\[ \sum (\text{Var\_Cost}_g + \text{Fuel\_Cost}_g) \times \text{MW}_{g,h} + \text{VOLL} \times \text{NSE}_h \]

Subject to:
- Supply = demand + non-served energy + network losses
- Operational constraints (ramps, startups)
- Reliability constraints (reserves)
- Network constraints (AC, DC, or transport)
- Contingencies?
- Environmental constraints?
If $X = (X_A, X_B)$, how would you represent this optimization problem graphically?
Optimization methods (3 of 3)

• Non Linear Programming **NLP**
  - Continuous variables
  - Non Linear constraints
  - Non linear objective function
    - Algorithms can compute local optimum, but how about the *global optimum*?
    - Very important limitations in the size of the problems

• Dynamic Programming:
  - Decisions are taken in several steps
Pattern search
Pattern search

Figure 3.7. Four iterations of Hooke-Jeeves search algorithm are marked with a filled circle.
DIRECT SEARCH SOLUTION OF NUMERICAL AND STATISTICAL PROBLEMS

\[ c. \]

If \( P, c B, i \), then \( B, = P, \) and \( S, = f(S, - 1) \). Otherwise, \( B, = B, - i \) and \( S, = g(S, - 1) \).

d. When for the first time \( S, = 0 \), the procedure stops; i.e., \( N = i \).

Direct search is distinguished from other numerical procedures by having a finite number of states which, without loss of generality, can be indexed by a set of integers. It includes, as special cases, those techniques for which \( h \) does not depend on \( B, - i \), such as (a) techniques which depend on inspection of all points of an arbitrarily determined set, or (b) Monte Carlo techniques which depend on a random selection of points in \( \mathbb{R} \).

It does not include methods which possess a continuum of states, such as those (e.g., Newton's method and methods of steepest ascent) which rely on the use of such tools as derivatives and power series approximations.

APPENDIX B

Pattern Search

An exact description of a pattern search routine, which has been applied, is given in the accompanying figures. A flow diagram for pattern search is given in Chart 1. The notation is self-explanatory. The sequence following the label (\( \cdots \)) is the basic iterative loop consisting of a pattern move followed by a set of exploratory moves. The sequence following the label (\( \cdots \)) is for an initial set of exploratory moves from a base point when a new pattern must be established. The sequence labeled (\( \cdots \)) controls the reduction of step size and termination of the search. The remaining charts (2--4) give details of the procedure. Explicitly the procedure is carried out by sequentially transforming a set of variables.

Descriptive flow diagram for pattern search
The “pattern search” method of Hooke & Jeeves does not require calculating a gradient. It includes the following steps:

- Define your space of candidate designs based on discrete options for the N design variables.

- Select an initial set of candidate designs to evaluate. This set consists of a “center point” design, plus designs that are n design options away from the center in each dimension.

- Calculate the cost of each of these design options.

- The lowest cost design becomes the new center point. If it is different from the previous center point, select a new set of designs that are n design options away from the center in each dimension, calculate their costs, and set the center point to the lowest cost point again.

- Repeat this until the center point is the same for subsequent steps. When this happens, replace n by n/2 (rounded up to a whole number), and continue the process.

- When n has a value of 1 and all the immediately adjacent design options are higher cost than the center point, the center point design is a local minimum. This is selected as the design, and the costs and specifications of this design are returned.

This pattern search is guaranteed to find a local minimum in the defined design space, but may not necessarily find the lowest-cost of all possible designs. In practice though, this method has performed well for the cases I have studied.
Optimization software

• There exist several Algebraic Modeling Languages specifically designed to develop optimization models
  – GAMS, OPL Studio, AMPL, MPL, AIMMS, LINGO…
  – Compact formulation
  – Data is separated from mathematical model
  – Formulation independent of the model size
  – Model independent of Solvers
  – Interfaces are not visual
Questions or comments?