2.830 Assignment 4 Solutions

Problem 1)

a) The new process control expert used the standard deviation of the 5 sample control Shewhart xbar data to calculate $C_p$ and $C_{pk}$, whereas he should have scaled by the square root of the sample size to have an approximation of the true population standard deviation.

b) The actual $C_p$ is $C_p = \frac{C_{p\text{-expert}}}{\sqrt{5}} = 0.75$. Similarly, $C_{pk} = \frac{C_{pk\text{-expert}}}{\sqrt{5}} = 0.59$.

c) A $C_{pk}$ of 0.59 means that

$$C_{pk} = \frac{\mu - SL_1}{3\sigma} = 0.59$$

$$(0.59)3\sigma = |\mu - SL_1| = 1.77\sigma$$

and a $C_p$ of 0.75 means that

$$C_p = \frac{USL - LSL}{6\sigma} = 0.75$$

$$|SL_2 - \mu| = 4.5\sigma - 1.77\sigma = 2.73\sigma$$

where $SL_1$ and $SL_2$ are the specification limits - but we do not know which is upper and which is lower, and we do not need to know to determine the fraction of parts out of specification.

The fraction of defective parts is the sum of the fraction of a normal distribution farther than 1.77 standard deviations from the mean on one side, and farther than 2.73 standard deviations from the mean on the other. Taking $z = 1.77$ and $z = 2.73$, from the tables we find the corresponding cumulative standard normal distribution function is 0.96164 and 0.99683, respectively. The fraction of defective parts is then

$$\text{Defective fraction} = (1 - 0.96164) + (1 - 0.99683) = 0.03836 + 0.00317 = 0.04153.$$
Problem 8-1.

\[ \mu_0 = 1050; \sigma = 25; \delta = 1\sigma; K = (\delta/2)\sigma = (1/2)25 = 12.5; H = 5\sigma = 5(25) = 125 \]

(a)  
MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM

The process signals out of control at observation 10. The point at which the assignable cause occurred can be determined by counting the number of increasing plot points. The assignable cause occurred after observation 10 – 3 = 7.

(b)  
\[ \hat{\sigma} = \frac{\bar{MR}_2}{d_2} = \frac{38.8421}{1.128} = 34.4345 \]  
equation 5-6.

No. The estimate used for \( \sigma \) is much smaller than that from the data.
The process signals out of control at observation 10. The assignable cause occurred after observation 10 – 3 = 7.
Problem 8-21 (8-19, 4th ed.).

\[ \lambda = 0.1, L = 2.7, \hat{\sigma} = 12.16, \text{CL} = \mu_0 = 950, \text{UCL} = 957.53, \text{LCL} = 942.47. \]

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA

Test Results for EWMA Chart of Ex8-7temp

TEST. One point beyond control limits.
Test Failed at points: 12, 13

Process is out of control at samples 8 (beyond upper limit, but not flagged on chart), 12 and 13.
\( \lambda = 0.4, L = 3, \sigma = 12.16, CL = \mu_0 = 950, UCL = 968.24, LCL = 931.76. \)

**MTB > Stat > Control Charts > Time-Weighted Charts > EWMA**

![EWMA Chart of Temperature Readings (Ex8-7temp)](image)

**Test Results for EWMA Chart of Ex8-7temp**

- TEST: One point beyond control limits.
- Test Failed at points: 70

With the larger \( \lambda \), the process is out of control at observation 70, as compared to the chart in the Exercise 21 (with the smaller \( \lambda \)) which signaled out of control at earlier samples.