Problem 1

A transmission line system incorporates two transmission lines with characteristic impedances of $Z_{01} = 100 \Omega$ and $Z_{02} = 50 \Omega$ as illustrated above. A voltage source is applied at the left end, $v(t) = 100 \cos (\pi \times 10^4 t)$. At this frequency, line 1 has length of $\frac{\lambda_1}{2} = 1$ meter and line 2 has length of $\frac{\lambda_2}{4} = 0.4$ meter, where $\lambda_1$ and $\lambda_2$ are the wavelengths along each respective transmission line.

The two transmission lines are connected by a series resistance $jX_z$ and the end of line 2 is loaded by impedance $Z_L = 50(1 + j) \Omega$. The voltage source is connected to line 1 through a source resistance $R_s = 50 \Omega$.

a) What are the speeds $c_1$ and $c_2$ of electromagnetic waves on each line?

$$\frac{f \lambda}{c} = \frac{f}{c} = \frac{50 \times 10^6}{3 \times 10^8} = \frac{50}{3} \text{ M}\mu\text{s}$$

$$\lambda_1 = \frac{c}{f_1} = \frac{3 \times 10^8}{50 \times 10^6} = 6 \text{ m}$$

$$\lambda_2 = \frac{c}{f_2} = \frac{3 \times 10^8}{10^7} = 30 \text{ m}$$

b) It is desired that $X_z$ be chosen so that the source current $i(t) = I_0 \cos (\pi \times 10^4 t)$ is in phase with the voltage source. What is $X_z$?

$$\frac{\lambda_2}{2} = 1 \text{ m}, \quad Z_0(\frac{\lambda_2}{2} - \frac{\lambda_1}{4}) = 1 \text{ m}, \quad Z(\frac{\lambda_2}{2} - \frac{\lambda_1}{4}) = 50(1 + j)$$

$$jX_z + 2Z(1-j) \text{ must be matched} \quad \Rightarrow X_z = 2Z = 2 \times 50 = 100 \Omega$$

c) For the value of $X_z$ in part (b), what is the peak amplitude $I_0$ of the source current $i(t)$?

Note that the value of $X_z$ itself is not needed to answer this question or part (d).

$$\frac{\bar{v}(\bar{z} = -\frac{\lambda_1}{4})}{2Z} = \frac{\bar{v}(\bar{z} = -\frac{\lambda_2}{4} - \frac{\lambda_1}{4})}{2Z} = \frac{25Z_0}{25Z_2}$$

$$i(t) = \frac{v(t)}{75Z_2} = \frac{4}{3} \cos (\pi \times 10^4 t) \Rightarrow I_0 = \frac{\bar{I}}{Z} = \frac{4}{3} \text{ Amperes}$$
Problem 2

A parallel plate waveguide is to be designed so that only TEM modes can propagate in the frequency range \(0 < f < 2 \text{ GHz}\). The dielectric between the plates has a relative dielectric constant of \(\varepsilon_r = 9\) and a magnetic permeability of free space \(\mu_0\).

a) What is the maximum allowed spacing \(d_{\text{max}}\) between the parallel plate waveguide plates?

b) If the plate spacing is 2.1 cm, and \(f = 10 \text{ GHz}\), what \(\text{TE}_n\) and \(\text{TM}_n\) modes will propagate?

\[
\omega_{co,n} = \frac{n \pi c}{d} = 2\pi f_{co,n} \Rightarrow f_{co,n} = \frac{n c}{2d}, \quad c = 10^8 \text{ m/s}
\]

\(n = 1\) lowest cut-off frequency

\[
f_{co,1} = \frac{c}{2d} > 2 \text{ GHz}
\]

\(c > 4 \times 10^9 \text{d}
\]

\(d < \frac{c}{4 \times 10^9}
\]

\(d < \frac{10^8}{4 \times 10^9}
\]

\(d < \frac{1}{40} \text{m} \Rightarrow d < 2.5 \text{ cm}
\]

b) \(f_{co,n} = \frac{n c}{2d} < 10 \text{ GHz}
\]

\(n < \frac{2d (10^9)}{c}
\]

\(n < 2 \cdot (0.21) \cdot 10^{10}
\]

\(n < 4.2
\]

**Propagating modes:** \(\text{TE}_1, \text{TE}_2, \text{TE}_3, \text{TE}_4, \text{TM}_1, \text{TM}_2, \text{TM}_3, \text{TM}_4\)
Problem 3

A transmission line of length $\ell$, characteristic impedance $Z_0 = 100\Omega$, and one-way time of flight $T = \frac{\ell}{c}$ is connected at $z = 0$ to a 100 volt DC battery through a series source resistance $R_s = 100\Omega$ and a switch. The $z = \ell$ end is loaded by a 300$\Omega$ resistor.

a) The switch at the $z = 0$ end has been closed for a very long time so that the system is in the DC steady state. What are the values of the positive and negative traveling wave voltage amplitudes $V_+(z-ct)$ and $V_-(z+ct)$?

\[
V_+(z,t) = V_+ = \frac{V_0}{\Delta S}, \quad V_-(z,t) = V_- = \frac{2.5V_0}{\Delta S}
\]

$\Delta S$ =

\[
V_+ = \frac{5V_0}{\Delta S}, \quad V_- = \frac{2.5V_0}{\Delta S}
\]

Part b, on the next page, to be handed in with your exam. Put your name at the top of the next page.
b) With the system in the DC steady state, the switch is suddenly opened at time $t = 0$.

i) Plot the positive and negative traveling wave voltage amplitudes, $V_+(z - ct)$ and $V_-(z + ct)$, as a function of $z$ at time $t = T/2$.

\[
\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300 - 100}{300 + 100} = \frac{200}{400} = \frac{1}{2}, \quad \Gamma_S = 1
\]

\[
V_+(z - cT/2) \quad V_-(z + cT/2)
\]

ii) Plot the transmission line voltage $v(z, t)$ as a function of $z$ at time $t = T/2$.

\[
v(z, t = T/2)
\]

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