Problem 1

a) Apply Integral form of Ampere’s Law:
\[2\pi r H_\phi = \int_{\phi=0}^{2\pi} \int_{r=0}^{R_2} J_z(r) r dr d\phi \rightarrow H = \frac{J_0 r^2}{3R_1} i_\phi\]

b) The total current going up the inside conductor must return going down in the outside conductor:
\[2\pi \int_{r=R_1}^{R_2} J_z(r) r dr = 2\pi R_2 |K| \rightarrow K = -\frac{J_0 R_2^2}{3R_2} i_x\]

Problem 1 Alternate Method:

\[R_1 < r < R_2\]
\[2\pi r H_\phi = \int_{\phi=0}^{2\pi} \int_{r=0}^{R_1} J_0 r R_1^2 \frac{r dr d\phi}{3r} \rightarrow H_\phi = \frac{J_0 R_2^2}{3r} \]
\[K_z = -H_\phi (r = R_2) = -\frac{J_0 R_2^2}{3R_2} \]

Problem 2

a) The boundary conditions at \(\phi = 0\) and \(\phi = \alpha\) determine the unknowns A and B
\[\Phi(\phi = 0) = 0 \rightarrow B = 0\]
\[\Phi(\phi = \alpha) = V_0 \rightarrow A = \frac{V_0}{\alpha}\]

b) The electric field is the negative gradient of the potential. Problem is best done in cylindrical coordinates:
\[E = -\nabla \Phi = -\frac{1}{r} \frac{\partial \Phi}{\partial \phi} i_\phi = \frac{V_0}{\alpha r} i_\phi\]

c) Use the boundary condition on the normal D field at the upper perfect conductor:
\[\sigma_{sf} = i_\phi \cdot [\varepsilon_0 E(\phi = \alpha^+) - \varepsilon E(\phi = \alpha^-)] = \frac{\varepsilon V_0}{\alpha r}\]

d) Since \(C = Q/V_0\) and the total charge is given by
\[Q = d \int_{r=R_1}^{R_2} \sigma_{sf} (r) dr = \frac{\varepsilon d}{\alpha} V_0 \ln \left(\frac{R_2}{R_1}\right) \rightarrow C = \frac{\varepsilon d}{\alpha} \ln \left(\frac{R_2}{R_1}\right)\]

Problem 3

a) By inspection \(\omega = 2\pi f = 2\pi \times 10^8 \rightarrow f = 10^8\) Hz

b) By inspection \(|k| = \sqrt{k_x^2 + k_z^2} = \pi \sqrt{1+3} = 2\pi \rightarrow \lambda = \frac{2\pi}{|k|} = 1\) m

c) \(c = f \lambda = 1\times 10^8\) m/s

d) From trigonometry \(k_x = \pi, \quad k_z = \pi \sqrt{3}, \quad \tan(\theta) = \frac{k_x}{k_z} = \frac{1}{\sqrt{3}} \rightarrow \theta = \frac{\pi}{6}\) radians, equal to 30 degrees.