1 Examples

1. [2]. For each of the following claims, state whether true or false and justify briefly. Assume standard form unless otherwise stated.

(a) \( N(A) \) is a subspace.

(b) A rank-deficient matrix always has a nullspace of dimension greater than zero.

(c) The system \( Ax = 0 \) has a nonzero solution if and only if the rows of \( A \) are linearly dependent.

(d) A nonempty polyhedron in standard form can contain at most \( \binom{n}{m} \) vertices, and there are LP instances for which this bound is tight.

(e) Degenerate BFSs result in a standard form system if and only if the system has redundant constraints.

(f) A BFS for a standard form system is degenerate if and only if multiple bases correspond to it.

(g) The polyhedron \( \{ x \in \mathbb{R}^n \mid Ax \leq 0 \} \) has either zero or one extreme point.

(h) The tableau columns corresponding to slack variables are precisely the columns of \( B^{-1} \).

(i) There exist multiple optimal solutions if and only if there exists a basis with some zero nonbasic reduced costs.

(j) Let \((x_B, x_N)\) be a BFS and \( \bar{v} \) be its reduced costs. For any other feasible solution \( y \), the cost difference between \( x \) and \( y \) is \( \bar{v}_N^T y_N \).

(k) Regardless of the pivoting rule employed, the simplex method will never cycle for a problem with one equality constraint.
(l) An algorithm for finding a solution to a set of linear inequalities implies an algorithm for solving an LP, and vice versa.

(m) The system \( Ax \leq b \) is feasible if and only if the rows of \( A \) are linearly independent.

(n) A system of linear inequalities is infeasible if and only if there exists a nonnegative linear combination of the inequalities which is inconsistent by itself.

(o) Dual degeneracy implies primal non-uniqueness.

(p) If the dual has multiple optimal solutions, then all optimal primal BFSs are degenerate.

2. Exercise 4.5, from [1].

References
