This problem set is due in recitation on Friday, December 8, 2006. The homework lab for this problem set will be held 7–9 P.M. on Wednesday, December 6, 2006, in room 32-155.

Reading: Chapter 34.

Both exercises and problems should be solved, but only the problems should be turned in. Exercises are intended to help you master the course material. Even though you should not turn in the exercise solutions, you are responsible for material covered in the exercises.

Mark the top of each sheet with your name, the course number, the problem number, your recitation section, the date and the names of any students with whom you collaborated.

You will often be called upon to “give an algorithm” to solve a certain problem. Your write-up should take the form of a short essay. A topic paragraph should summarize the problem you are solving and what your results are. The body of the essay should provide the following:

1. A description of the algorithm in English and, if helpful, pseudo-code.
2. At least one worked example or diagram to show more precisely how your algorithm works.
3. A proof (or indication) of the correctness of the algorithm.
4. An analysis of the running time of the algorithm.

Remember, your goal is to communicate. Full credit will be given only to correct solutions which are described clearly. Convoluted and obtuse descriptions will receive low marks.

Exercise 9-1. Do Exercise 34.2-1 on page 982 in CLRS.

Problem 9-1. Subgraph Isomorphism

Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are said to be isomorphic if there exists a bijection $\pi : V_1 \rightarrow V_2$, providing a one-to-one correspondence between the vertices of the two graphs, such that $(u, v) \in E_1$ if and only if $(\pi(u), \pi(v)) \in E_2$. A graph $H = (V_H, E_H)$ is a subgraph of $G = (V_G, E_G)$ if $V_H \subseteq V_G$ and $E_H \subseteq E_G$.

Consider the subgraph-isomorphism problem: given two graphs $G$ and $H$, determine whether $G$ is isomorphic to a subgraph of $H$. Prove that the subgraph-isomorphism problem is NP-complete.
Problem 9-2. Linear Difference Constraints

Motivated by the fun experience of Quiz 2, and also hoping to impress the 6.046 staff, you set your sights on tackling a harder, more ambitious problem. In particular, you decide to explore the relationship between solving systems of linear difference constraints, and the CLIQUE problem described in lecture.

Consider the decision problem \( \text{DIFF} \) of determining whether a system of linear difference constraints on \( b \)-bit integers is feasible. Formally, let \( S_b \) denote the set of integers between \([-2^b, 2^b-1]\). Consider a set \( C \) of \( m \) constraints, \( C = \{ C^{(1)}, C^{(2)}, \ldots, C^{(m)} \} \). Each \( C^{(k)} \) is a triple \( C^{(k)} = (p^{(k)}, q^{(k)}, w^{(k)}) \), with \( p^{(k)}, q^{(k)} \in \{1, 2, \ldots, n\} \) and \( w^{(k)} \in S_{b+1} \). Each triple \( C^{(k)} \) represents the constraint \( x_{q^{(k)}}^{(k)} - x_{p^{(k)}}^{(k)} \leq w^{(k)} \). For any input \( \langle C, n \rangle \), let \( A(C, n) \subseteq S_n^b \) be the set of tuples \((x_1, x_2, \ldots, x_n)\) such that, for all \( 1 \leq k \leq m \), we have \( x_{q^{(k)}}^{(k)} - x_{p^{(k)}}^{(k)} \leq w^{(k)} \). In other words, \( A(C, n) \) is the set of all solutions to the given system of linear difference constraints. Then define \( \text{DIFF} \) to be the language of feasible systems of linear difference constraints, i.e.,

\[
\text{DIFF} = \{ \langle C, n \rangle : A(C, n) \neq \emptyset \}.
\]

(a) Suppose that you find a polynomial-time reduction from \( \text{DIFF} \) to CLIQUE. What consequences, if any, does this result have for the question of P vs. NP? Will the 6.046 staff be impressed by your result? Explain why or why not.

(b) Suppose instead that you find a polynomial-time reduction from CLIQUE to \( \text{DIFF} \). What consequences, if any, does this result have for the question of P vs. NP? Will the 6.046 staff be impressed by your result? Explain why or why not.

(c) **Bonus Part**: This part is optional. Solving this part does not exempt you from solving the previous parts of this question. A complete and correct solution will, however, impress the 6.046 staff, will have a positive impact on your grade, and may come with a monetary reward.

For both parts (a) and (b), find a polynomial-time reduction matching the one described in the problem statement. Prove the correctness of your results.