Lecture 6
Feedback and Control
Feedback and Control

Feedback is pervasive in natural and artificial systems.

Feedback is a process by which an action or response is controlled by the consequences of the action or response. In natural systems, feedback is often used to maintain homeostasis, such as in the case of a car's steering system, where the driver's input is compared to the actual position of the car to adjust the steering wheel accordingly.

For example, when driving a car, the driver turns the steering wheel to stay centered in the lane. The desired position is the intended position of the car, and the actual position is the current position of the car. The difference between the desired and actual positions is used to adjust the steering wheel, ensuring the car stays centered in the lane.

Turn steering wheel to stay centered in the lane.

Desired position -> driver -> car -> actual position
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Feedback is useful for regulating a system’s behavior, as when a thermostat regulates the temperature of a house.
Feedback and Control

Concentration of glucose in blood is highly regulated and remains nearly constant despite episodic ingestion and use.
Feedback and Control

Motor control relies on feedback from pressure sensors in the skin as well as proprioceptors in muscles, tendons, and joints.

Try building a robotic hand to unscrew a lightbulb!

Shadow Dexterous Robot Hand (Wikipedia)
Feedback and Control

Today’s goal: use systems theory to gain insight into how to control a system.
Feedback and Control

Steering a car: controlling its lateral position.

Algorithm: steer left when right of center and vice versa.
Feedback and Control

Steering a car: controlling its lateral position.

Algorithm: steer left when right of center and vice versa.
Feedback and Control

Steering a car: controlling its lateral position.

Algorithm: steer left when right of center and vice versa.

straight ahead?

steer left

steer left
Feedback and Control

Steering a car: controlling its lateral position.

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Feedback and Control

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Feedback and Control

Steering a car: controlling its lateral position.

Algorithm: steer left when right of center and vice versa.
Feedback and Control

Bad algorithm $\rightarrow$ poor performance.

Here we get persistent oscillations!

straight ahead?
steer right
steer right
steer right
straight ahead?
steer left
steer left
Feedback and Control

The persistent oscillation is a mode of the system.

Control almost always involves feedback.

Feedback $\rightarrow$ cyclic paths $\rightarrow$ persistent outputs (modes).

Studying the behaviors of modes helps you design control algorithms.
Example

Guide a person to walk down the center of a lane by telling how far to move left/right on next step.
Example

First consider how the person processes motion commands.
Example

First consider how the person processes motion commands.

- 1. command rightward +1
First consider how the person processes motion commands.

1. command rightward +1
Example

First consider how the person processes motion commands.

1. command rightward +1
2. command rightward +1
Example

First consider how the person processes motion commands.

1. command rightward +1
2. command rightward +1
Example

First consider how the person processes motion commands.

1. command rightward +1
2. command rightward +1
3. command rightward 0
Example

First consider how the person processes motion commands.

1. command rightward +1
2. command rightward +1
3. command rightward 0
Example

First consider how the person processes motion commands.

1. command rightward +1
2. command rightward +1
3. command rightward 0
4. command rightward 0
Example

First consider how the person processes motion commands.

1. command rightward $+1$
2. command rightward $+1$
3. command rightward $0$
4. command rightward $0$
Example

First consider how the person processes motion commands.

1. command rightward +1
2. command rightward +1
3. command rightward 0
4. command rightward 0

Motion commands “accumulate” to give person’s position.

\[ \frac{p}{c} = \frac{r}{1-r} \]
Example

Now add feedback:

- Observe current position $p[n]$.  
- Tell person to move rightward by $-p[n]$.  

○
Example

Now add feedback:

- Observe current position $p[n]$.
- Tell person to move rightward by $-p[n]$. 

1. see $-1$: command rightward $+1$
Now add feedback:

- Observe current position $p[n]$.
- Tell person to move rightward by $-p[n]$.

1. see $-1$: command rightward $+1$
Example

Now add feedback:

- Observe current position $p[n]$.
- Tell person to move rightward by $-p[n]$.

1. see $-1$: command rightward $+1$
2. see $0$: command rightward $0$
### Example

Now add feedback:

- Observe current position $p[n]$.
- Tell person to move rightward by $-p[n]$.

<table>
<thead>
<tr>
<th>1. see $-1$: command rightward $+1$</th>
<th>2. see $0$: command rightward $0$</th>
</tr>
</thead>
</table>
Example

Now add feedback:

- Observe current position $p[n]$.
- Tell person to move rightward by $-p[n]$.

1. see $-1$: command rightward $+1$
2. see $0$: command rightward $0$
3. see $0$: command rightward $0$
Example

Now add feedback:

- Observe current position $p[n]$.
- Tell person to move rightward by $-p[n]$.

1. see $-1$: command rightward +1
2. see 0: command rightward 0
3. see 0: command rightward 0
Example

Now add feedback:

• Observe current position $p[n]$.
• Tell person to move rightward by $-p[n]$.

1. see $-1$: command rightward $+1$
2. see $0$: command rightward $0$
3. see $0$: command rightward $0$

Easy.
Example

What if you cannot see the current position, but only the prior position.

1. see previous \((-1)\) $\rightarrow$ command $+1$
Example

What if you cannot see the current position, but only the prior position.

1. see previous \((-1)\) → command +1
Example

What if you cannot see the current position, but only the prior position.

1. see previous (−1) → command +1
2. see previous (0) → command 0
What if you cannot see the current position, but only the prior position.

1. see previous \((-1)\) → command \(+1\)

2. see previous \((0)\) → command 0

1. see previous \((-1)\) → command +1
Example

What if you cannot see the current position, but only the prior position.

1. see previous \((-1)\) → command \(+1\)
2. see previous \((0)\) → command \(0\)
3. see previous \((+1)\) → command \(-1\)
Example

What if you cannot see the current position, but only the prior position.

1. see previous $(−1)$ → command $+1$
2. see previous $(0)$ → command $0$
3. see previous $(+1)$ → command $−1$
Example

What if you cannot see the current position, but only the prior position.

- 1. see previous \((-1)\) $\rightarrow$ command $+1$
- 2. see previous \((0)\) $\rightarrow$ command $0$
- 3. see previous \((+1)\) $\rightarrow$ command $-1$
- 4. see previous \((+1)\) $\rightarrow$ command $-1$

1. see previous \((-1)\) $\rightarrow$ command $+1$
2. see previous \((0)\) $\rightarrow$ command $0$
3. see previous \((+1)\) $\rightarrow$ command $-1$
4. see previous \((+1)\) $\rightarrow$ command $-1$
Example

What if you cannot see the current position, but only the prior position.

1. see previous \((-1)\) → command \(+1\)
2. see previous \((0)\) → command \(0\)
3. see previous \((+1)\) → command \(-1\)
4. see previous \((+1)\) → command \(-1\)
What if you cannot see the current position, but only the prior position.

1. see previous (−1) → command +1
2. see previous (0) → command 0
3. see previous (+1) → command −1
4. see previous (+1) → command −1
5. see previous (0) → command 0
Example

What if you cannot see the current position, but only the prior position.

1. see previous \((-1)\) → command \(+1\)
2. see previous \((0)\) → command \(0\)
3. see previous \((+1)\) → command \(-1\)
4. see previous \((+1)\) → command \(-1\)
5. see previous \((0)\) → command \(0\)
Example

What if you cannot see the current position, but only the prior position.

1. see previous (−1) → command +1
2. see previous (0) → command 0
3. see previous (+1) → command −1
4. see previous (+1) → command −1
5. see previous (0) → command 0

Introducing delay can destabilize a control system.

Key issue in biological and artificial control systems.
Feedback and Control

Make model to understand destabilizing effect of delay.

\[ + \alpha R_1 - R_1 R X Y \]

\[ S \]

\[ \frac{R}{1 - R} \]

controller

person

sensor
Check Yourself: Which is True?

1. \[ \frac{Y}{X} = \frac{\alpha R}{1 - R} \]
2. \[ \frac{Y}{X} = \frac{\alpha R}{1 - R + \alpha R^2} \]
3. \[ \frac{Y}{X} = \frac{\alpha R}{1 + R - \alpha R^2} \]
4. \[ \frac{Y}{X} = \frac{\alpha R}{1 - R} - \alpha R \]
Check Yourself: Which is True?

\[
\frac{Y}{X} = \frac{\alpha R}{1 - R}
\]

1. \[
\frac{Y}{X} = \frac{\alpha R}{1 - R}
\]

2. \[
\frac{Y}{X} = \frac{\alpha R}{1 - R + \alpha R^2}
\]

3. \[
\frac{Y}{X} = \frac{\alpha R}{1 + R - \alpha R^2}
\]

4. \[
\frac{Y}{X} = \frac{\alpha R}{1 - R} - \alpha R
\]
To find the modes of this system, put denominator of system functional in the form \((1 - p_0R)(1 - p_1R)\), then the modes are at \(z = p_0\) and \(z = p_1\).

\[
\frac{\gamma}{X} = \frac{\alpha R}{1 - R + \alpha R^2} = \frac{\alpha R}{(1 - p_0R)(1 - p_1R)}
\]

\[
1 - R + \alpha R^2 = 1 - (p_0 + p_1)R + p_0 p_1 R^2
\]

\[
p_0, p_1 = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - \alpha}
\]
If \( \alpha \) is small, the modes occur at \( z \approx \alpha \) and \( z \approx 1 - \alpha \).

\[
p_0, p_1 = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - \alpha} = \frac{1}{2} \left(1 \pm \sqrt{1 - 4\alpha}\right) \approx \frac{1}{2} \left(1 \pm (1 - 2\alpha)\right) = 1 - \alpha, \alpha
\]

Little feedback: person stands almost still because mode 2 shrinks so slowly.
Feedback and Control

Make model to understand destabilizing effect of delay.
Feedback and Control: Persistent Responses

As $\alpha$ increases, the modes move toward each other and collide at $z = \frac{1}{2}$ when $\alpha = \frac{1}{4}$.

$$p_0, p_1 = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - \alpha} = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - \frac{1}{4}} = \frac{1}{2}, \frac{1}{2}$$

Persistent responses decay. Position of person $\to 0$. 

![Diagram showing modes and z-plane with $\alpha = \frac{1}{4}$]
Example

What if you cannot see the current position, but only the prior position.

1. see previous (−1) → command +1
2. see previous (0) → command 0
3. see previous (+1) → command −1
4. see previous (+1) → command −1
5. see previous (0) → command 0

Q: What is \( \alpha \) in this example?
If $\alpha > 1/4$, the modes become complex.

$$p_0, p_1 = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - \alpha} = \frac{1}{2} \pm j\sqrt{\alpha - \left(\frac{1}{2}\right)^2}$$

Complex modes $\rightarrow$ oscillations.
Feedback and Control: Persistent Responses

We saw this behavior when we were guiding the person.

1. see previous (−1) → command +1
2. see previous (0) → command 0
3. see previous (+1) → command −1
4. see previous (+1) → command −1
5. see previous (0) → command 0
Feedback and Control: Persistent Responses

The pole-zero diagram shows why we got oscillations. We tried to correct the error all at once: \( \alpha = 1 \). Too much feedback!

Try using less feedback.
Change the control algorithm, command smaller steps: \( \alpha = 0.5 \).

1. see previous \((-1) \rightarrow \text{command } +1/2\)
Change the control algorithm, command smaller steps: $\alpha = 0.5$.

1. see previous $(-1) \rightarrow$ command $+1/2$
Change the control algorithm, command smaller steps: $\alpha = 0.5$.

1. see previous $(-1) \rightarrow$ command $+1/2$
2. see previous $(0) \rightarrow$ command $0$
Change the control algorithm, command smaller steps: \( \alpha = 0.5 \).

1. see previous \((-1)\) → command \(+1/2\)
2. see previous \((0)\) → command \(0\)
Feedback and Control: Persistent Responses

Change the control algorithm, command smaller steps: \( \alpha = 0.5 \).

1. see previous (−1) → command +1/2
2. see previous (0) → command 0
3. see previous (+1/2) → command −1/4
Change the control algorithm, command smaller steps: $\alpha = 0.5$.

1. see previous $(-1)$ → command $+1/2$
2. see previous $(0)$ → command $0$
3. see previous $(+1/2)$ → command $-1/4$
Change the control algorithm, command smaller steps: $\alpha = 0.5$.

1. see previous (−1) → command +1/2
2. see previous (0) → command 0
3. see previous (+1/2) → command −1/4
4. see previous (+1/4) → command −1/8
Change the control algorithm, command smaller steps: $\alpha = 0.5$.

1. see previous $(-1) \rightarrow$ command $+1/2$
2. see previous $(0) \rightarrow$ command $0$
3. see previous $(+1/2) \rightarrow$ command $-1/4$
4. see previous $(+1/4) \rightarrow$ command $-1/8$
Feedback and Control: Persistent Responses

Change the control algorithm, command smaller steps: \( \alpha = 0.5 \).

1. see previous (−1) → command +1/2
2. see previous (0) → command 0
3. see previous (+1/2) → command −1/4
4. see previous (+1/4) → command −1/8
5. see previous (+1/8) → command −1/16
Change the control algorithm, command smaller steps: \( \alpha = 0.5 \).

1. see previous (−1) → command +1/2
2. see previous (0) → command 0
3. see previous (+1/2) → command −1/4
4. see previous (+1/4) → command −1/8
5. see previous (+1/8) → command −1/16
Original “guide the person problem” had no sensor delay.

Illustrate how the pole position(s) depend on $\alpha$. 
Original “guide the person problem” had no sensor delay.

\[
Y = \frac{\alpha R}{1 - R} E = \frac{\alpha R}{1 - R} X - Y
\]

\[
\frac{Y}{X} = \frac{\alpha R}{1 - (1 - \alpha)R}
\]
As the closed-loop pole approaches $z = 0$, the response decays faster toward zero, placing the person in the center of the lane.
Feedback and Control: Summary

Feedback is useful for controlling systems.
Feedback $\rightarrow$ cyclic signal paths $\rightarrow$ persistent outputs (modes).
Pole-zero plots help visualize how modes depend on amount of feedback.