6.003 Signals and Systems
Exam 1 Outline

While outline this is not intended to be a complete and exhaustive checklist, these are tasks that you are expected to be able to do.

Basic mathematical skills

• Solving a linear differential equation with constant coefficients by finding the homogeneous and particular solutions, know how to handle repeated roots in the characteristic polynomial, and use initial values to determine the correct constants in your solution.
• Using basic physics (8.01, 8.02) or common real-life examples (such as a bank) to model a simple physical system (mass-and-spring, RLC, etc.) as a differential equation.
• Solving a linear difference equation with constant coefficients by finding the homogeneous and particular solutions, know how to handle repeated roots in the characteristic polynomial, and use initial values to determine the correct constants in your solution.
• Understanding the meaning/definition/consequences of linear difference/differential equations and linear systems.
• Partial fractions
• Long division of rational expressions (or equivalent technique)
• Taylor series and when it is a good approximation (is the Taylor expansion of \( \sqrt{x - 1} \) useful to approximate \( \sqrt{1000 - 1} \)?)

Representations of systems

• Converting between any of following ways of expressing a system:
  – Verbal or physical description of the system
    An uncurable disease has been discovered in a large town. If a person catches a disease one day, (s)he becomes contagious the next day and stays contagious forever. Each contagious person causes one more person to catch the disease each day. We wish to model this situation as a system where the input signal \( x[n] \) is the number of people in the town that caught the disease from outside the town on day \( n \), and the output \( y[n] \) is the number of total people with the disease (whether contagious or not yet) on day \( n \).
  – Difference equation
    \[ y[n] = y[n - 1] + y[n - 2] + x[n] \] (1)
– System function $Y/X$ expressed as a rational function

$$\frac{Y}{X} = \frac{1}{(1 - R - R^2)}$$  \hspace{1cm} (2)

– System function $Y/X$ expressed in terms of partial fractions

$$\frac{Y}{X} = \frac{\phi/\sqrt{5}}{1 - \phi R} - \frac{(1 - \phi)/\sqrt{5}}{1 - (1 - \phi)R}, \quad \phi = (1 + \sqrt{5})/2$$  \hspace{1cm} (3)

– System function $Y/X$ expressed as a power series

$$\frac{Y}{X} = 1 + R + 2R^2 + 3R^3 + 5R^4 + 8R^5 + \ldots$$  \hspace{1cm} (4)

– Impulse response

$$y_{\text{impulse}}[n] = \frac{1}{\sqrt{5}}(\phi^{n+1} - (1 - \phi)^{n+1})$$  \hspace{1cm} (5)

– Samples of the impulse response

$$1, 1, 2, 3, 5, 8, \ldots$$  \hspace{1cm} (6)

– Block diagram

![Block diagram](image)

– Pole-zero diagram

![Pole-zero diagram](image)
• Identifying and use a well-suited method (among the many we have learned) to convert from any of the above to any other of the above

• Using system functions and block diagrams to your advantage by separating a large system into a cascade of smaller systems you already understand (for example, \( \frac{1}{1-2R+R^2} \) is \( \frac{1}{1-R} \ast \frac{1}{1-R} \), a cascade of two accumulators

**Modes of a system**

• Identifying the modes of a system given any of the above representations and give the closed form of each mode (\( G^n \) for some complex \( G \) you must find)

• Knowing whether modes are exponential/sinusoidal, growing/decaying/constant in amplitude

• Drawing an equivalent block-diagram of the system as a sum of its individual modes

• Giving examples of signals that are guaranteed to excite a particular mode.

• Giving examples of signals are guaranteed to suppress a particular mode.

**Approximating CT systems as DT systems**

• Using the Forward-Euler, Backward-Euler, and trapezoidal (”leapfrog”) methods

• Estimating suitable/reasonable time steps to use for a particular physical system

**Feedback and control**

• Understanding and justifying the use of feedback as a technique

• Constructing feedback loops to stabilize given systems

• Deriving the overall system function \( Y/X \) of such a sytem with a feedback controller, leaving gains in the system (in particular, the ones in the controller) as parameters

• Identifying and understanding the poles and zeros of a system both before and after adding feedback loops

• Understanding how those poles and zeros move in the \( z \)-plane as the gain parameters are varied (called a root-locus diagram), and the stability consequences of various values

• Understanding the \( z \)-plane and what types of responses are characteristic of poles on different parts of the plane

• P vs. PD controllers and when P controllers are not sufficient to make a system stable

• Understanding the limitations of pole-zero cancellation (for example, when the system is not initially at rest)