Warmups

1. Ranking frequencies
   For a second-order, pole-only system with \( Q = 1.5 \), order the following frequencies from lowest to highest:
   - the oscillation frequency \( \omega_d \),
   - the natural frequency \( \omega_0 \),
   - the maximum-gain frequency \( \omega_{\text{max}} \), which is the frequency to which the system is most sensitive (i.e. for which it has the largest gain).
   
   Draw a rough pole–zero diagram for the system, and mark the three frequencies on the positive imaginary axis.

2. Phase
   For a second-order system with poles at \(-1\) and \(-4\) (and no zeros), find the frequency at which the phase is \(-90^\circ\), using any method except for the vector method. Then illustrate and confirm that result using the vector method.

3. Multiple representations
   Each part of this question gives partial information about a second-order system without zeros, for example an LRC low-pass circuit. For each system, use the information to estimate \( Q \) and \( \omega_0 \) and to sketch the impulse response and pole–zero diagram. If it is possible from the partial information, give scales on the real and imaginary axes of pole–zero diagrams, and give scales on the time axis of the impulse responses. [No need to repeat the information given, e.g. if you are given \( Q \), there’s no need to estimate it.]
   
   a. \( H(s) = 1/(s^2 + s + 1) \)
   b. Pole plot (all poles are on the real axis):
      
      \[ \begin{array}{c}
      \times && \times \\
      -18 & \text{Re } p & -2
      \end{array} \]
   c. \( Q = 0.5 \)
   d. In the following impulse response, the dashed vertical lines mark one-second intervals.
      
      \[ \text{Impulse response} \]
   e. \( Q = 5 \)
4. Maximum gain
For each system, find the maximum-gain frequency $\omega_{\text{max}}$, which is the frequency to which the system is most sensitive (has the largest gain).

a. $\frac{1}{1 + s + s^2}$

b. $\frac{s}{1 + s + s^2}$

c. $\frac{s^2}{1 + s + s^2}$

Compare the $\omega_{\text{max}}$ for these systems and explain qualitatively any similarities or differences.

Problems
5. Feedback
How do you build a reliable oscillator out of analog components? This feedback circuit is a precursor to Hewlett and Packard's founding patent where they solved that hard problem:

![Feedback circuit diagram]

a. With $R = 1 \, \text{k}\Omega$ and $C = 1 \, \mu\text{F}$, sketch the pole locations as the gain $K$ varies from 0 to $\infty$, showing a scale for the real and imaginary axes. Find the $K$ for which the system is barely stable, and label your sketch with that information. What is the system's oscillation period at this $K$?

b. How do your results change if each $R$ is increased to $10 \, \text{k}\Omega$?

6. Two ways to combine two RC circuits
You might have wondered about any differences between cascading two RC circuits with and without an intervening unity-gain buffer. In this question you work out the answer. Here are diagrams of the two possibilities:

![RC circuit diagrams]

For each system, use the state-variable method to find the differential equation connecting $V_{\text{out}}$ and $V_{\text{in}}$. Compare the respective system functions, impulse responses, $Q$'s, and whatever else seems interesting. Discuss qualitatively any similarities and differences.
7. **Hours**

While our primary goal in designing homework assignments is that these exercises should be educational, we know that they take time. Please help us determine how reasonable the workload in 6.003 is by estimating how many hours you spent during the past week working on this homework assignment. Feel free also to comment on these problems.