Lecture 13
CT Frequency Response and Bode Plots
Substituting $\frac{1}{s}$ for $\mathcal{A}$ and $s$ for $\mathcal{D}$ converts a CT system functional into its corresponding **system function** $H(s)$.

$$H(s) = \left. \frac{Y}{X} \right|_{\mathcal{A} \leftarrow \frac{1}{s}, \mathcal{D} \leftarrow s}$$

Poles (or zeros) are roots of the denominator (or numerator) polynomial of $H(s)$.

$$H(s) = \frac{Y}{X} = \frac{100s}{s^2 + 2s + 100}$$
The system function can be used to determine response of a system to exponential inputs.

If \( x(t) = e^{s_0 t} \) (for all time) then \( y(t) = H(s_0)e^{s_0 t} \) (for all time).

Exponentials are eigenfunctions of systems that can be represented by linear differential equations with constant coefficients.

The eigenvalue associated with the eigenfunction \( e^{s_0 t} \) is the value of the system function \( H(s) \) at \( s = s_0 \).
The value of $H(s_0)$ at a point $s = s_0$ can be determined graphically using vector analysis.

Factor the numerator and denominator of the system function to make poles and zeros explicit.

$$H(s_0) = K \frac{(s_0 - z_0)(s_0 - z_1)(s_0 - z_2)\cdots}{(s_0 - p_0)(s_0 - p_1)(s_0 - p_2)\cdots}$$

Each factor in the numerator/denominator corresponds to a vector from a zero/pole (here $z_0$) to $s_0$, the point of interest in the $s$-plane.
The value of $H(s_0)$ is the product of the vectors that correspond to zeros divided by the product of the vectors that correspond to poles.
The magnitude of \( H(s_0) \) is the product of the magnitudes of the vectors associated with the zeros divided by the product of the magnitudes of the vectors associated with the poles.

\[
|H(s_0)| = K \frac{\prod_{q=1}^{Q} (s_0 - z_q)}{\prod_{p=1}^{P} (s_0 - p_p)} = |K| \frac{\prod_{q=1}^{Q} |s_0 - z_q|}{\prod_{p=1}^{P} |s_0 - p_p|}
\]
The angle of $H(s_0)$ is the sum of the angles of the vectors associated with the zeros minus the sum of the angles of the vectors associated with the poles.

$$\angle H(s_0) = \angle \left( K \frac{\prod_{q=1}^{Q} (s_0 - z_q)}{\prod_{p=1}^{P} (s_0 - p_p)} \right) = \angle K + \sum_{q=1}^{Q} \angle (s_0 - z_q) - \sum_{p=1}^{P} \angle (s_0 - p_p)$$

The angle of $K$ can be 0 or $\pi$ for systems described by linear differential equations with constant, real-valued coefficients.
The frequency response of a system is given by $H(j\omega)$.

If $x(t) = \cos \omega_0 t$ then

$$y(t) = |H(j\omega)| \cos(\omega_0 t + \angle(H(j\omega))).$$

The frequency response is a complex-valued function of $\omega$. The magnitude gives the gain of the system for each frequency. The angle gives the phase.
Frequency Response: \( H(s) |_{s \leftarrow j\omega} \)

\[ H(s) = s - z_1 \]

- \( s \)-plane
  - \( \sigma \)
  - \( \omega \)

- \( |H(j\omega)| \)
  - \( -5 \) to \( 5 \)

- \( \angle H(j\omega) \)
  - \( -\pi/2 \) to \( \pi/2 \)
Frequency Response: $H(s)\big|_{s \leftarrow j\omega}$

$H(s) = s - z_1$
Frequency Response: \( H(s) \big| _{s \leftarrow j\omega} \)

\[
H(s) = s - z_1
\]

\( s \)-plane

<table>
<thead>
<tr>
<th>( H(j\omega) )</th>
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<tbody>
<tr>
<td>5</td>
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<tr>
<td>-5</td>
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\( \angle H(j\omega) \)

<table>
<thead>
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<th>( \angle H(j\omega) )</th>
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<tr>
<td>( \pi/2 )</td>
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<tr>
<td>-5</td>
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<td>-( \pi/2 )</td>
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\( \omega \)

\( \sigma \)
Frequency Response: \( H(s) \mid_{s \leftarrow j\omega} \)

\[ H(s) = s - z_1 \]

\[ |H(j\omega)| \]

\[ \angle H(j\omega) \]
Frequency Response: $H(s)|_{s \leftarrow j\omega}$

$$H(s) = s - z_1$$

$s$-plane

$|H(j\omega)|$

$\angle H(j\omega)$
**Frequency Response:**  $H(s)|_{s \leftarrow j\omega}$

\[ H(s) = s - z_1 \]

$s$-plane

$|H(j\omega)|$

$\angle H(j\omega)$
Frequency Response: $H(s) \mid_{s \leftarrow j\omega}$

$$H(s) = s - z_1$$

$s$-plane

$\omega$

$\sigma$

$|H(j\omega)|$

$\angle H(j\omega)$
Frequency Response: \[ H(s) \big|_{s \leftarrow j\omega} \]

\[ H(s) = s - z_1 \]

\[ |H(j\omega)| \]

\[ \angle H(j\omega) \]
Frequency Response: \( H(s)|_{s \leftarrow j\omega} \)

\[ H(s) = s - z_1 \]

\[ s \rightarrow j\omega \]

\( s \)-plane

\[ |H(j\omega)| \]

\( \angle H(j\omega) \)

\( \pi/2 \)

\( -\pi/2 \)
Frequency Response: \( H(s) \mid_{s \leftarrow j\omega} \)

\[
H(s) = s - z_1
\]

\[
|H(j\omega)|
\]

\[
\angle H(j\omega)
\]
Frequency Response: $H(s)|_{s \leftarrow j\omega}$

$H(s) = s - z_1$
Frequency Response: $H(s)|_{s \leftarrow j\omega}$

$$H(s) = s - z_1$$

$s$-plane

$|H(j\omega)|$

$\angle H(j\omega)$
Frequency Response: \( H(s)|_{s \leftarrow j\omega} \)

\[
H(s) = \frac{9}{s - p_1}
\]

\( s \)-plane

\( \omega \)

\( \sigma \)

\( |H(j\omega)| \)

\( \angle H(j\omega) \)
Frequency Response: \( H(s) \vert_{s \leftarrow j\omega} \)

\[
H(s) = \frac{9}{s - p_1}
\]

\[
|H(j\omega)|
\]

\[
\angle H(j\omega)
\]
**Frequency Response:**  \( H(s)|_{s \leftarrow j\omega} \)

\[
H(s) = \frac{9}{s - p_1}
\]
Frequency Response: \( H(s) \left|_{s \leftarrow j\omega} \right. \)

\[
H(s) = \frac{9}{s - p_1}
\]
Frequency Response: $H(s)_{s \leftarrow j\omega}$

\[ H(s) = \frac{9}{s - p_1} \]

\[ |H(j\omega)| \]

\[ \angle H(j\omega) \]
**Frequency Response:** \( H(s)|_{s \leftarrow j\omega} \)

\[
H(s) = \frac{9}{s - p_1}
\]
Frequency Response: \( H(s) \mid_{s \leftarrow j\omega} \)

\[
H(s) = \frac{9}{s - p_1}
\]

![Graph showing frequency response](image)
Frequency Response: \( H(s)|_{s \leftarrow j\omega} \)

\[
H(s) = \frac{9}{s - p_1}
\]

\( s\)-plane

\( \omega \)

\( \sigma \)

\( \pi/2 \)

\(-\pi/2\)
Frequency Response: \( H(s) \mid_{s \leftarrow j\omega} \)

\[
H(s) = \frac{9}{s - p_1}
\]

\[
|H(j\omega)|
\]

\[
\angle H(j\omega)
\]
**Frequency Response:** \( H(s) \mid_{s \leftarrow j\omega} \)

\[
H(s) = \frac{9}{s - p_1}
\]

- **Frequency Response Plot:**
  - \( |H(j\omega)| \)
  - \( \angle H(j\omega) \)
Frequency Response: \( H(s) \big|_{s \leftarrow j\omega} \)

\[
H(s) = \frac{9}{s - p_1}
\]
Frequency Response: \( H(s) \rvert_{s \leftarrow j\omega} \)

\[ H(s) = \frac{9}{s - p_1} \]

- \( \omega \)
- \( \sigma \)
- \( s \)-plane

\[ |H(j\omega)| \]

\[ \angle H(j\omega) \]

-5 \( \pi/2 \)
0 \( -\pi/2 \)
5

-5
0
5
Frequency Response: \( H(s)|_{s\leftarrow j\omega} \)

\[
H(s) = 3 \frac{s - z_1}{s - p_1}
\]
**Frequency Response:** $H(s)|_{s \leftarrow j\omega}$

$$H(s) = 3 \frac{s - z_1}{s - p_1}$$

![Diagram showing the frequency response](image)

- **Magnitude Response:** $|H(j\omega)|$
  - Values range from 0 to 5.
- **Phase Response:** $\angle H(j\omega)$
  - Values range from $-\pi/2$ to $\pi/2$. 

The diagram illustrates the behavior of the system's response in the frequency domain.
Frequency Response: \( H(s)|_{s\leftarrow j\omega} \)

\[
H(s) = 3 \frac{s - z_1}{s - p_1}
\]

\( s \)-plane

\[ |H(j\omega)| \]

\[ \angle H(j\omega) \]
Frequency Response:  \( H(s) \mid_{s \leftarrow j\omega} \)

\[
H(s) = 3 \frac{s - z_1}{s - p_1}
\]
Frequency Response: $H(s)|_{s \leftarrow j\omega}$

$$H(s) = 3\frac{s - z_1}{s - p_1}$$

$\omega$

$s$-plane

$\sigma$

$\pi/2$

$-\pi/2$
Frequency Response: $H(s)|_{s \leftarrow j\omega}$

\[ H(s) = 3 \frac{s - z_1}{s - p_1} \]

$|H(j\omega)|$

$\angle H(j\omega)$
Frequency Response: $H(s)|_{s \leftarrow j\omega}$

$$H(s) = 3 \frac{s - z_1}{s - p_1}$$
Frequency Response: \( H(s) \|_{s \leftarrow j\omega} \)

\[
H(s) = 3 \frac{s - z_1}{s - p_1}
\]
Frequency Response: \( H(s) \big|_{s \to j\omega} \)

\[ H(s) = 3 \frac{s - z_1}{s - p_1} \]

\( s \)-plane

\( \sigma \quad \omega \)

\[ |H(j\omega)| \]

\[ \angle H(j\omega) \]
Frequency Response: $H(s)|_{s \leftarrow j\omega}$

\[
H(s) = \frac{3s - z_1}{s - p_1}
\]
Frequency Response: \( H(s)|_{s \leftarrow j\omega} \)

\[
H(s) = 3 \frac{s - z_1}{s - p_1}
\]
Frequency Response: \( H(s) \) \( \mid_{s \leftarrow j\omega} \)

\[
H(s) = 3 \frac{s - z_1}{s - p_1}
\]

[Graph of frequency response with modulus and angle plots]
Check Yourself

Could the phase plots of any of the systems represented by the following pole-zero plots be equal to each other? [caution: this could be a trick question]
Check Yourself

1. $\omega$ $\pi$

2. $\omega$ $\pi$

3. $\omega$ $\pi$

4. $\omega$ $\pi$
Check Yourself

1. \[ -1 \]
2. \[ \begin{array}{c}
1 \hline
-1
\end{array} \quad \text{if } K < 0 \]
3. \[ \begin{array}{c}
2 \hline
-1
\end{array} \]
4. \[ \begin{array}{c}
2 \hline
-1
\end{array} \]
From Frequency Response to Bode Plot

The magnitude of $H(j\omega)$ is a product of magnitudes.

$$|H(j\omega)| = |K| \frac{Q}{p} \prod_{q=1}^{Q} |j\omega - z_q|$$

The angle of $H(j\omega)$ is a sum of angles.

$$\angle H(j\omega) = \angle K + \sum_{q=1}^{Q} \angle (j\omega - z_q) - \sum_{p=1}^{P} \angle (j\omega - p_p)$$

The angle of $K$ can be 0 or $\pi$ for systems described by linear differential equations with constant, real-valued coefficients.
From Frequency Response to Bode Plot

The log of the magnitude is a sum of logs.

\[ |H(j\omega)| = |K| \frac{\prod_{q=1}^{Q} |j\omega - z_q|}{\prod_{p=1}^{P} |j\omega - p_p|} \]

\[ \log |H(j\omega)| = \log |K| + \sum_{q=1}^{Q} \log |j\omega - z_q| - \sum_{p=1}^{P} \log |j\omega - p_p| \]

\[ \angle H(j\omega) = \angle K + \sum_{q=1}^{Q} \angle (j\omega - z_q) - \sum_{p=1}^{P} \angle (j\omega - p_p) \]
Bode Plot: Isolated Zero

The low-frequency asymptote is \( \log |H(j\omega)| = \log |z_1| \).

\[
H(s) = s - z_1 , \ z_1 < 0 \quad \left| \frac{H(j\omega)}{|z_1|} \right| \quad [\text{log scale}]
\]

\[
\lim_{\omega \to 0} |H(j\omega)| = |z_1|
\]
The high-frequency asymptote is $\log |H(j\omega)| = \log \omega$.

$$H(s) = s - z_1 \ , \ z_1 < 0$$

$$\lim_{\omega \to 0} |H(j\omega)| = |z_1|$$

$$\lim_{\omega \to \infty} |H(j\omega)| = \omega$$
Bode Plot: Isolated Zero

The two asymptotes are a good approximation to $\log|H(j\omega)|$.

$$H(s) = s - z_1, \quad z_1 < 0$$

$$\lim_{\omega \to 0} |H(j\omega)| = |z_1|$$

$$\lim_{\omega \to \infty} |H(j\omega)| = \omega$$
Bode Plot: Isolated Zero

The low-frequency asymptote is $\angle H(j\omega) = 0$.

$$H(s) = s - z_1, \ z_1 < 0$$

$$\lim_{\omega \to 0} \angle H(j\omega) = 0$$
The high-frequency asymptote is $\angle H(j\omega) = \pi/2$.

$$H(s) = s - z_1 \quad , \quad z_1 < 0$$
Bode Plot: Isolated Zero

Straight-line approximation to $\angle H(j\omega)$.

$$H(s) = s - z_1 \quad , \quad z_1 < 0$$

$$\lim_{\omega \to 0} \angle H(j\omega) = 0$$

$$\lim_{\omega \to \infty} \angle H(j\omega) = \pi / 2$$
The low-frequency asymptote is \( \log |H(j\omega)| = \log |1/p_1| \).

\[
H(s) = \frac{1}{s - p_1}, \quad p_1 < 0
\]

\[
|p_1||H(j\omega)| \quad \text{[log scale]}
\]

\[
\lim_{\omega \to 0} |H(j\omega)| = |1/p_1|
\]
The high-frequency asymptote is \(\log |H(j\omega)| = \log 1/\omega\).

\[
H(s) = \frac{1}{s - p_1}, \quad p_1 < 0
\]

\[
\lim_{\omega \to 0} |H(j\omega)| = |1/p_1|
\]

\[
\lim_{\omega \to \infty} |H(j\omega)| = 1/\omega
\]
Bode Plot: Isolated Pole

The two asymptotes are a good approximation to \( \log |H(j\omega)| \).

\[
H(s) = \frac{1}{s - p_1}, \quad p_1 < 0
\]

\[
\lim_{\omega \to 0} |H(j\omega)| = \frac{1}{|p_1|}
\]

\[
\lim_{\omega \to \infty} |H(j\omega)| = \frac{1}{\omega}
\]
The low-frequency asymptote is $\angle H(j\omega) = 0$.

$$H(s) = \frac{1}{s - p_1}, \quad p_1 < 0$$

$$\lim_{\omega \to 0} \angle H(j\omega) = 0$$
The high-frequency asymptote is \( \angle H(j\omega) = -\pi/2 \).

\[ H(s) = \frac{1}{s - p_1}, \quad p_1 < 0 \]

\[ \lim_{\omega \to 0} \angle H(j\omega) = 0 \]

\[ \lim_{\omega \to \infty} \angle H(j\omega) = -\pi/2 \]
Bode Plot: Isolated Pole

Straight-line approximation to $\angle H(j\omega)$.

$$H(s) = \frac{1}{s - p_1} , \quad p_1 < 0$$

$\lim_{\omega \to 0} \angle H(j\omega) = 0$

$\lim_{\omega \to \infty} \angle H(j\omega) = -\pi/2$
Check Yourself

\[ H_1(s) = \frac{1}{s + 1} \quad \text{and} \quad H_2(s) = \frac{10}{s + 10} \]

The Bode magnitude plot for \( H_2(s) \) can be obtained from that for \( H_1(s) \) by

1. shifting it horizontally
2. scaling it horizontally
3. shifting and scaling it horizontally
4. shifting and scaling both horizontally and vertically
5. none of the above
Bode Plot: More Complicated

\[ H(s) = \frac{s}{(s + 1)(s + 10)} \]
Bode Plot: More Complicated

\[ H(s) = \frac{s}{(s + 1)(s + 10)} \]

s-plane

\[ \log \left| \frac{s}{s + 1} \right| \]

\[ \log \left| \frac{1}{s + 10} \right| \]
Bode Plot: More Complicated

\[ H(s) = \frac{s}{(s + 1)(s + 10)} \]
Bode Plot: More Complicated

\[ H(s) = \frac{s}{(s + 1)(s + 10)} \]
Bode Plot: More Complicated

\[ H(s) = \frac{s}{(s + 1)(s + 10)} \]
Bode Plot: More Complicated

\[ H(s) = \frac{s}{(s + 1)(s + 10)} \]
Bode Plot: More Complicated

\[ H(s) = \frac{s}{(s + 1)(s + 10)} \]
Bode Plot: More Complicated

\[ H(s) = \frac{s}{(s + 1)(s + 10)} \]
Bode Plot: dB

\[ H(s) = \frac{10s}{(s + 1)(s + 10)} \]

s-plane

\[ \log |H(j\omega)| \]

\[ \angle H(j\omega) \]
Bode Plot: dB

\[ H(s) = \frac{10s}{(s + 1)(s + 10)} \]
$$H(s) = \frac{10s}{(s + 1)(s + 10)}$$

Bode Plot: dB

$$|H(j\omega)|[dB] = 20 \log_{10} |H(j\omega)|$$

$$\angle H(j\omega)$$
$H(s) = \frac{10s}{(s + 1)(s + 10)}$

$|H(j\omega)|[\text{dB}] = 20 \log_{10}|H(j\omega)|$

$\angle H(j\omega)$
The straight-line approximations are surprisingly accurate.

\[ H(j\omega) = \frac{1}{j\omega + 1} \]

- \( X \rightarrow 20\log_{10} X \) dB
  - 1 dB
  - \( \sqrt{2} \approx 3 \) dB
  - 2 dB
  - 10 dB
  - 100 dB

\[ H(j\omega) = \frac{1}{j\omega + 1} \]

- Phase: \( 0.1 \text{ rad} \) (6°)
Check Yourself

Without using a calculator (other than the one in your head) determine

1. the frequency that is 6dB below 1kHz
2. the amplitude that is 10 dB above 1
3. the value of 500 in dB
4. how many dB/octave correspond to a slope of 20dB/decade
Frequency Response of a High-$Q$ System

The magnitude of the frequency response of a high-$Q$ system is peaked.

$$H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$
Frequency Response of a High-$Q$ System

The magnitude of the frequency response of a high-$Q$ system is peaked.

\[ H(s) = \frac{1}{1 + \frac{1}{Q \omega_0} + \left(\frac{s}{\omega_0}\right)^2} \]
Frequency Response of a High-$Q$ System

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The magnitude of the frequency response of a high-$Q$ system is peaked.

\[
H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}
\]

\[
\frac{s}{\omega_0} - \text{plane}
\]

\[
\log |H(j\omega)|
\]

\[
\frac{\omega}{\omega_0} \quad \text{[log scale]}
\]
Check Yourself

Estimate the peak value of the magnitude function as a function of $Q$ assuming $Q$ is large (e.g., $Q > 3$).

$$H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$
Frequency Response of a High-$Q$ System

As $Q$ increases, the width of the peak narrows.

\[ H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left( \frac{s}{\omega_0} \right)^2} \]

\[ \log |H(j\omega)| \]
**Frequency Response of a High-\(Q\) System**

As \(Q\) increases, the width of the peak narrows.

\[
H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}
\]

![Diagram](image)
Frequency Response of a High-\(Q\) System

As \(Q\) increases, the width of the peak narrows.

\[
H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}
\]
Frequency Response of a High-$Q$ System

As $Q$ increases, the width of the peak narrows.

$$H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$
Frequency Response of a High-Q System

As $Q$ increases, the width of the peak narrows.

\[
H(s) = \frac{1}{1 + \frac{1}{Q \omega_0} + \left( \frac{s}{\omega_0} \right)^2}
\]

\[
\log |H(j\omega)|
\]

$s/\omega_0$-plane

$\log |H(j\omega)|$
Estimate the “3dB bandwidth” of the peak.

Let $\omega_l$ (or $\omega_h$) represent the lowest (or highest) frequency for which the magnitude is greater than the peak value divided by $\sqrt{2}$. The 3dB bandwidth is then $\omega_h - \omega_l$. 
Frequency Response of a High-$Q$ System

As $Q$ increases, the phase changes more abruptly with $\omega$.

$$H(s) = \frac{1}{1 + \frac{1}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$
Frequency Response of a High-$Q$ System

As $Q$ increases, the phase changes more abruptly with $\omega$.

$$H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left( \frac{s}{\omega_0} \right)^2}$$
Frequency Response of a High-$Q$ System

As $Q$ increases, the phase changes more abruptly with $\omega$.

$$H(s) = \frac{1}{1 + \frac{1}{Q \omega_0} s + s^2}$$
Frequency Response of a High-$Q$ System

As $Q$ increases, the phase changes more abruptly with $\omega$.

$$H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$
Frequency Response of a High-$Q$ System

As $Q$ increases, the phase changes more abruptly with $\omega$.

\[ H(s) = \frac{1}{1 + \frac{1}{Q\omega_0} s + \left(\frac{s}{\omega_0}\right)^2} \]
Check Yourself

Estimate the change in phase that occurs over the 3dB bandwidth.

\[ H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \]

[Diagram showing the \( s/\omega_0 \)-plane with points labeled and a plot of \( \angle |H(j\omega)| \) against \( \omega/\omega_0 \) on a log scale.]
Multiple Representations of CT Systems

Verbal descriptions: preserve the underlying physics.

Differential equations: mathematically compact.

\[ \dot{y}(t) = x(t) + py(t) \]

Block diagrams: illustrate signal flow paths.

Operator representations: analyze systems as polynomials.

\[ (1 - pA)Y = AX \]

Pole-Zero diagrams: represent factors of system functional.

System functions: represent systems as polynomials in \( s \).

Bode plots: quickly sketch the frequency response.