6.003 Quiz 2 Sanity Check Questions

(These ARE NOT intended to be exhaustive, nor represent the level of difficulty of the quiz. These are expected to be EASY sanity-check questions; if they are not, make sure to ask us about them.)

Impulse/step responses

(a) Given the impulse response of a system, how can you compute the step response? Given the step response, how can you compute the impulse response?

(b) What are the impulse and step responses corresponding to the following system functionals/functions? (Assume the systems are causal.)

- $A, \frac{1}{A}, A - 1, \frac{A}{1+A}, \frac{A^2}{(1+A)(1-2A)}$
- $\frac{1}{s}, \frac{1}{s^2}, \frac{1}{s^3}, s - 1, \frac{s-1}{s^2}, \frac{1}{(s-1)^2}, \frac{1}{(s-1)(s+2)}, \frac{1}{s}, \frac{1}{s^2}, \frac{1}{s(s-1)(s+2)}, \frac{s+1}{(s-1)(s+2)}$

Poles and zeros

(a) Given a CT system described by a LCCDE at initial rest, which has poles at $-0.5 + j, -0.5 - j$ and a zero at 1. Is the system stable? How many modes does it have?

(b) Same as (a), with an additional pole at $-1$. Is the system stable? How many modes does it have?

(c) Same as (b), except the pole at $-1$ is a double pole. Is the system stable? How many modes does it have?

(d) Construct a pole-zero plot for a system whose impulse response is oscillatory and neither growing nor decaying.

(e) A system has impulse response $h(t) = \delta(t) + 2e^{-3t}u(t)$. Draw its pole-zero plot.

Eigenfunctions

(a) Given a CT system described by a LCCDE at initial rest. Is each of the following signals guaranteed to be an eigenfunction of the system?

- $e^{2t}$
- $5e^{j\pi t}$
- $e^{2t} + 5e^{j\pi t}$
- $\cos(\pi t)$
- $\cos(\pi t) + jsin(\pi t)$
- $\delta(t)$, the unit impulse
- $u(t)$, the unit step
- 2 (careful!)
(b) Suppose the system in (a) has system function \( H(s) = 1 + s \). For each of the signals in (a) that you determined to be eigenfunctions, what is its corresponding eigenvalue?

(c) For the system in (b), given that the input is \( x(t) = \cos(\pi t) + 3 \), what is the output \( y(t) \)?

**Bode plots**

(a) Draw the Bode magnitude and phase plots for a single pole at \(-p, p > 0\), and for a single zero at \(-z, z > 0\). Use the canonical forms \( \frac{1}{1+s/p} \) and \( 1 + s/z \) for the system functions.

(b) Draw the Bode magnitude and phase plots for a single pole at the origin, \( H(s) = 1/s \), and for a single zero at the origin, \( H(s) = s \).

(c) Draw the Bode and phase plots for two complex conjugate poles, using the canonical system function \( \frac{1}{s - \omega_0^2 + j\omega_0} \).

(d) Given the Bode and phase plots for \( H(s) \), what do the Bode plots for \( KH(s) \) look like, for some positive real constant \( K \)?

(e) Given the Bode and phase plots for \( H(s) \), what do the Bode plots for \( \frac{1}{H(s)} \) look like?

(f) Given the Bode and phase plots for \( H(s) \) and for \( G(s) \), what do the Bode plots for \( H(s)G(s) \) look like?

**Miscellaneous**

(a) Write \( 5e^{j\pi/3t} \) in rectangular/Cartesian form. Write \( \frac{\sqrt{\pi+1}j\pi}{-\sqrt{\pi+1}} \) in polar form.

(b) Evaluate \( j^{1/2} \) and \( j^{1/3} \) (how many distinct answers should there be in each case?)

(c) What is the frequency in Hz of \( \sin(\frac{\pi}{2} t) \)?

(d) Evaluate \( \sum_{n=0}^{\infty} \frac{p^n t^n}{n!} \) for an arbitrary constant \( p \).

(e) Plot the result of applying the system with system function \( s \) to the input \( f(t) = \begin{cases} 0, & t < 0 \\ t, & 0 < t < 3 \\ 2, & 3 < t < 5 \\ 0, & t > 5 \end{cases} \).

(f) Plot the result of applying the operator \( A \) to the function \( f(t) = u(t) + \delta(t + 3) \).

(g) Write down a differential equation for each of the following:

- The system with system functional \( \frac{A^2 - 9}{(1+A)^2(1-2A)} \).
- The system with system function \( \frac{s^2 - 9}{(1+s)^2(1-2s)} \).
- The second-order pole-only system with \( Q = .1, \omega_0 = \pi/4 \), and a gain at frequency 0 of \( H(j0) = 3 \).