Warmup 1

$s(t)$

speech signal

$f_{\text{max}} = 4 \text{kHz}$

O&W p. 583

$\sum_{k=-\infty}^{\infty} s(t-kT)$

$S_m(t)$

modulated speech signal

$1 \text{ MHz desired}$

$\sum_{k=-\infty}^{\infty} s(t-kT) \xrightarrow{\text{FT}} \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} S(w - \frac{2\pi k}{T}) = P(jw)$

$S_m(t) = s(t) \cdot p(t)$

$s_m(jw) = S(jw) \ast P(jw)$

$\therefore \text{ we want } \frac{1}{T} = f_m = 1 \text{ MHz} \implies T = 1 \mu s$

We don’t modulate radio signals by multiplying by this impulse train because an impulse train is not a physically achievable signal. Also, since the impulse train contains a large number of high frequency signals, it is less practical than a bandlimited sinusoid.
Medium Problem 2

\[ \chi(t) \xrightarrow{\frac{1}{2s+1}} \ldots \xrightarrow{\frac{1}{2s+1}} y(t) \]

Anti-aliasing filter = \( H(jw) \)

Slope = -20 dB/decade

\( f_c = 20 \text{kHz} \) \(<\) corner frequency of filter

\( f_s = 44 \text{kHz} \).

\[ \tau = \frac{1}{2\pi f_c} = 8 \mu s \]

\[ H(jw) = \frac{1}{(2s+1)^n} \]

\[ |H(jw)| = \left( \frac{1}{(2w^2+1)} \right)^n \]

We want no aliasing (-80 dB) at 20 kHz due to the higher frequency sampled signal. Since it has a corner freq. of 44 kHz - 20 kHz = 24 kHz, we want

\[ |H(jw)| = \left( \frac{1}{(2w^2+1)} \right)^n = \frac{1}{10^4} (-80 \text{dB}) \quad @ \quad \omega = 2\pi f = 2\pi (24k) \]

\[ 10^4 = (2w^2+1)^{n/2} = (2.46)^{n/2} \]

\[ \log 4 = \frac{n}{2} \log(2.46) \quad \Rightarrow \quad n = 20 \]

(or \( n = 21 \))

(continued on next page)
Medium Problem 2 cont'd.

For $n = 20$, @ 20kHz, each filter contributes
-3dB of gain (definition of corner freq.)

\[ |H(j\omega)|_{@20kHz} = 20 \text{ (-3dB)} = -60 \text{ dB} \]

This can be confirmed using the analytic formula
for $|H(j\omega)|$ on the previous page.
Harder Problem 3

a) \( z(t) \) can equal \( x(t) \) if \( X(j\omega) \) is bandlimited so that \( X(j\omega) = 0 \) outside the range \( |\omega| < \frac{\pi}{2T} \).

b) \( z(t) \) never equals \( x(t) \) if \( x(t) \) has frequency components near \( \omega = 0 \).

c) \( z(t) \) can equal \( x(t) \) if \( X(j\omega) \) is bandlimited so that \( X(j\omega) = 0 \) outside the range \( |\omega| < \frac{\pi}{T} \).
Harder problem 4

a) Since $Y(jw)$ is bandlimited, $y(t)$ spans infinite time.
   \[ x(t) = 1 + 2 \cos(\frac{\pi}{3}t) + \cos(\frac{\pi}{2}t) \]

   $X(jw) = 2\pi \delta(w) + (2\pi \delta(w + \frac{\pi}{3}) + 2\pi \delta(w - \frac{\pi}{3})) + \pi (\delta(w - \frac{\pi}{2}) + \delta(w + \frac{\pi}{2}))$

   By linearity, the output cannot contain any frequency that is not in the input. \(\therefore\) The system is NOT linear.

b) No. Same argument as in part a).

cont'd. on next page.
c) From part a), we know

\[ X(j\omega) = 2\pi s(\omega) + 2\pi \left(s(\omega - \frac{\pi}{3}) + s(\omega + \frac{\pi}{3})\right) + \pi \left(s(\omega - \frac{\pi}{5}) + s(\omega + \frac{\pi}{5})\right) \]

\[ \therefore \text{Its magnitude is given in Figure A.} \]

\[ y(t) \xrightarrow{\mathcal{F}} Y(j\omega) \]

\[ Z_2 = y(t - \frac{\pi}{2}) \xrightarrow{\mathcal{F}} e^{-j\omega \frac{\pi}{2}} Y(j\omega) \quad \text{Time Shift Property} \]

Since \( |e^{-j\omega \frac{\pi}{2}}| = 1 \), \( |Z_2(j\omega)| = |Y(j\omega)| \) (unchanged)

\[ \therefore |Z_2(j\omega)| \text{ given in Figure C.} \]

\[ Z_3 = x(t) * y(t) \xrightarrow{\mathcal{F}} Z_3(j\omega) = \frac{1}{2\pi} X(j\omega) * Y(j\omega) \quad \text{Figure G} \]

By graphical convolution.

\[ Z_4(t) = x(t) * y(t) \xrightarrow{\mathcal{F}} Z_4(j\omega) = X(j\omega) Y(j\omega) \quad \text{Figure B} \]

Since \( x(\omega) \) and \( y(\omega) \) only overlap @ \( \omega = 0 \).

9) Since \( Y(j\omega) \) is real-valued for all \( \omega \), the phase \( \theta \)

\[ Z_5(j\omega) = e^{-j\omega \frac{\pi}{2}} Y(j\omega) \text{ comes only from the } e^{-j\omega \frac{\pi}{2}} \text{ term.} \]
h) \( z_6(t) = y(t) e^{j\frac{\pi}{2}t} \xrightarrow{FT} Y(j(w - \frac{\pi}{2})) \)

\( \therefore \) The phase is unchanged. It is only shifted in frequency by \( \frac{\pi}{2} \). Since \( Y(jw) \) is real for all \( w \), its phase is zero everywhere. \( Z_6(jw) \) has the same phase.

i) \( z_7 = \frac{dy(t)}{dt} \xrightarrow{FT} jw \ Y(jw) \)

The phase of \( jw \) is always 90° since it is purely imaginary. The phase of \( Y(jw) = 0° \) since it is purely real.

\( \therefore \) The phase of \( Z_7(jw) = 90° \) for all \( w \).