Warmup problems

1. Modulation
   What impulse train would shift speech to 1 MHz (a typical AM frequency)?
   Why don’t we modulate radio signals by multiplying by this impulse train (instead of by the sine wave)?

Medium problems

2. How many RC filters
   Use a cascade of $n$ identical first-order low-pass filters to implement an antialiasing filter for audio signals. Assume that the sampling rate is 44 kHz, as it is for CDs, and that the corner frequencies of the filters are at 20 kHz. Determine the minimum $n$ so that all frequency components that alias to frequencies between 0 and 20 kHz are attenuated by at least 80 dB (a factor of $10^4$ in amplitude). What is the magnitude of the frequency response of the cascade at 20 kHz?

Harder problems

3. Scrambled Transmission
   Speech is to be split into and transmitted over multiple frequency bands as follows

   $\begin{align*}
   x(t) \rightarrow & H(j\omega) \rightarrow E/M \text{ wave} \rightarrow \sum_{k=-\infty}^{\infty} \delta(t - kT) \\
   & \text{Ideal LPF} \rightarrow c(t) \rightarrow z(t)
   \end{align*}$

   where $T$ is a positive, real-valued constant and $H(j\omega)$ is defined below.

   $H(j\omega) = \begin{cases} 
   1 & 0 \leq \omega < \frac{3\pi}{2T} \\
   0 & \frac{3\pi}{2T} \leq \omega < \frac{2\pi}{T} \\
   1 & \frac{2\pi}{T} \leq \omega < \frac{3\pi}{T} \\
   0 & \frac{3\pi}{T} \leq \omega < \frac{7\pi}{2T} \\
   \end{cases}$

   a. Let $c(t) = \cos(2\pi t/T)$. If $x(t)$ is bandlimited, is it possible to choose parameters for the ideal lowpass filter so that you can reconstruct $x(t)$ (i.e. make $z(t) = x(t)$)? If so, what is the maximum frequency that can be in $X(j\omega)$? If not, briefly explain why not.

   b. Repeat part (a) for $c(t) = \cos(4\pi t/T)$.

   c. Repeat part (a) for $c(t) = \cos(2\pi t/T) + \cos(4\pi t/T)$.
4. Fourier Transforms
Consider two continuous-time signals: \( x(t) \) shown below
\[
x(t) = 1 + 2 \cos \left( \frac{\pi}{3} t \right) + \cos \left( \frac{\pi}{2} t \right)
\]
and \( y(t) \), whose Fourier transform is shown below. \( Y(j\omega) \) is real-valued for all \( \omega \).

a. Is it possible to design a linear, time-invariant system whose input is \( x(t) \) and whose output is \( y(t) \)?
b. Is it possible to design a linear, time-invariant system whose input is \( y(t) \) and whose output is \( x(t) \)?
c. Which if any of the following plots shows the magnitude of the Fourier transform of \( z_1(t) = x(t) \)?

\[ \begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D} \\
\text{E} \\
\text{F} \\
\text{G} \\
\end{array} \]

\[
\begin{array}{c}
0 & \frac{\pi}{3} & \frac{\pi}{2} \\
\frac{-\pi}{12} & \frac{\pi}{12} \\
\frac{5\pi}{12} & \frac{7\pi}{12} \\
\frac{-\pi}{6} & \frac{\pi}{6} \\
\frac{\pi}{6} & \frac{\pi}{2} \\
\end{array}
\]

d. Repeat part (c) for \( z_2(t) = y(t - \frac{\pi}{2}) \).
e. Repeat part (c) for \( z_3(t) = x(t)y(t) \).
f. Repeat part (c) for \( z_4(t) = x(t) \ast y(t) \).
g. Sketch the angle of the Fourier transform of \( z_5 = y(t - \frac{\pi}{2}) \).
h. Sketch the angle of the Fourier transform of \( z_6 = y(t)e^{j\frac{\pi}{2}} \).
i. Sketch the angle of the Fourier transform of \( z_7 = \frac{d}{dt} y(t) \).
5. **Hours**

While our primary goal in designing homework assignments is that these exercises should be educational, we know that they take time. Please help us determine how reasonable the workload in 6.003 is by estimating how many hours you spent during the past week working on this homework assignment. Feel free also to comment on these problems.