Modulation

- Wireless communication application
- Impulse function definition and properties
- Fourier Transform of Impulse, Sine, Cosine
- Picture analysis using Fourier Transforms

Copyright © 2007 by M.H. Perrott & C. G. Sodini
All rights reserved.
Motivation for Modulation

- Modulation is used to change the frequency band of a signal
  - Enables RF communication in different frequency bands
  - Used in cell phones, AM/FM radio, WLAN, cable TV, ....
  - Note: higher frequencies lead to smaller antennas
The Fourier Transform as a Tool

- Communication signals are often non-periodic
- Fourier Transforms allow us to do modulation and filtering analysis using pictures

\[ x(t) \Leftrightarrow X(f) \]

- Where:

\[ x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df \]

\[ X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \]
Definition of the Impulse Function

• An impulse of area $A$ at time $t_o$ is denoted as:

$$A\delta(t - t_o)$$

• Impulses are defined in terms of their properties
  - Area:

$$\int_{-\infty}^{\infty} A\delta(t - t_o) = A$$

  - Fourier Transform:

$$A\delta(t - t_o) \Leftrightarrow Ae^{-j2\pi ft_o}$$

  - Sampling and convolution properties
    • Shown on the next two slides
Sampling Property of Impulses

- **Multiplication** of an impulse and a continuous function leads to *scaling* of the original impulse
  - The scale factor corresponds to the *sample value* of the continuous function at the impulse location

\[ A\delta(t - t_o)y(t) = Ay(t_o)\delta(t - t_o) \]
Fourier Transform of Cosine Wave

- Two real impulses in frequency needed for cosine in time

\[ x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} \, df \]

\[ = \int_{-\infty}^{\infty} \frac{K}{2} \left( \delta(f+f_0) + \delta(f-f_0) \right) e^{j2\pi ft} \, df \]

\[ = \frac{K}{2} \left( e^{-j2\pi f_0 t} + e^{j2\pi f_0 t} \right) \]

\[ = K \cos(2\pi f_0 t) \]

\[ \begin{array}{c}
\text{K} \cos(2\pi f_0 t) \\
\uparrow
\end{array} \] \[ \begin{array}{c}
\frac{K}{2} \left( \delta(f+f_0) + \delta(f-f_0) \right)
\end{array} \]
Fourier Transform of Sine Wave

- Two imaginary impulses in frequency needed for sine in time

\[ x(t) = \int_{-\infty}^{\infty} \frac{jk}{2} \left( \delta(f+f_o) - \delta(f-f_o) \right) e^{j2\pi ft} df \]

\[ = \frac{jk}{2} \left( e^{-j2\pi f_0 t} - e^{j2\pi f_0 t} \right) \]

\[ = \frac{K}{j2} \left( -e^{-j2\pi f_0 t} + e^{j2\pi f_0 t} \right) \]

\[ = K \sin(2\pi f_0 t) \]
Convolution Property of Impulses

- **Convolution** of an impulse and a function leads to **shifting** and **scaling** of the original function
  - The shift value corresponds to the location of the impulse
  - The scale factor corresponds to the area of the impulse
- Convolution is not limited to impulses
  - 6.003 will explore this in great detail

\[ A\delta(t - t_o) \ast y(t) = Ay(t - t_o) \]
AM Modulation (Transmitter)

- **AM** stands for *amplitude* modulation
  - Frequency and phase modulation are also commonly used
- Key operation is to *multiply* (i.e. *mix*) an input signal with a cosine (or sine) wave
  - This leads to an oscillating waveform whose amplitude varies according to the input signal
- Multiplication in time leads to convolution in frequency

\[ x(t)y(t) \Leftrightarrow X(f) \ast Y(f) \]
Fourier Transform Allows Picture Analysis

- Input signal is shifted to higher frequency band
AM Demodulation (Receiver)

- Input signal is shifted to lower and higher frequency bands
  - Want baseband portion

\[ y(t) = 2\cos(2\pi f_o t) \]
Impact of Frequency Offset

- Baseband signal is corrupted!
  - Filtering cannot fix this
Summary

• The impulse function is an important concept for Fourier Transform analysis
  - Fourier Transforms of cosines and sines consist of impulses
  - Defined in terms of its properties
    • Area, Multiplication (sampling), Convolution

• The Fourier Transform allows picture analysis of modulation and filtering
  - Modulation *shifts* in frequency (convolution with impulses)
  - Filtering *multiplies* in frequency

• Filtering in next lecture
  - Design of filters in Matlab (for Lab exercises)