1. We solve these problems pictorially, using the convolution technique explained in lecture 4 slides 10-11. Note that the components around DC interfere and cancel or partially cancel in (b) and (c), because there the input signal is real but the modulating signal is +/- imaginary.

a.

b.
2. The lowest frequency is inside the (flat) passband of the filter, and the phase shift is given as zero, so it remains unchanged except for the overall doubling of amplitude. The middle frequency is half-way down the roll-off, which we can assume is linear from the picture, so its amplitude is not doubled. The higher frequency is completely outside the filter’s range and roll-off, so it disappears.

\[ y(t) = 2 \cos(2\pi(900kHz)\cdot t) + 3 \sin(2\pi(2MHz)\cdot t) \]

3. We use the same pictorial techniques as above.

a.

A:

B:
b. The original signal has been split in half, so we'll need to "paste" the two halves together. We can do this by mixing with a 500Hz signal, as shown below. Then, low-pass filtering will keep only the -500Hz-500Hz range, leaving us with the original restored input signal, up to a scale factor. The scale factor cannot be resolved by choice of frequency. (In practice, there would be some distortion pasting together the sliced halves, because the filters' imperfect roll-offs would leave less-than-sharp edges.)