1. We need some more practice with Fourier Series and Transforms. These questions are very similar to the problems you had in the homework and miniquiz. Please try to do them without referring to the HW1 solutions.

(a) Plot the Fourier Series $\hat{X}_n$ for

$$x(t) = 2 - \cos(w_0 t) + \sin(2w_0 t) + 4 \cos(4w_0 t)$$

(b) Find $X_n$ for the signal in Figure 1.

![Figure 1: Periodic signal $x(t)$](image1.png)

(c) Consider the signal $x(t)$ in Figure 2.

![Figure 2: Non-periodic signal $x(t)$](image2.png)
i. Find $X(f)$.
ii. Find $|X(f)|$ and $\Phi(f)$, where $X(f) = |X(f)|e^{j\Phi(f)}$.
iii. Plot the magnitude of $X(f)$ over a reasonable frequency range to see its key characteristics.

2. Modulation and Filtering

(a) Consider the system and input $X(f)$ given in Figure 3.

![Modulation System](image)

Figure 3: Modulation System

i. If $f_o = 5500\text{Hz}$ and $s_1(t) = 5000\text{Hz}$, what should $s_2(t)$ and $f_c$ be so that $y(t) = x(t)$. Plot the spectrum for $M(f)$, $A(f)$, and $D(f)$.
ii. If $f_o = 6000\text{Hz}$, $s_1(t) = 6000\text{Hz}$, $s_2(t) = 5500\text{Hz}$, and $f_c = 0$, what kind of system must I implement at $y(t)$ to recover the original signal? Plot the spectrum for $M(f)$, $A(f)$, and $D(f)$ for this case.

(b) Draw frequency response sketches that represent the ideal lowpass, highpass, bandpass, and notchpass filters. Why can we never achieve these idealities? What do we do instead?

(c) In lecture, we briefly explored practical lowpass filters. We will now consider a possible implementation. Assume we want a trapezoidal response like $H(f)$ shown in Figure 4. This is no more than convolving two pulses in the frequency domain.

![Creating a practical lowpass filter](image)

Figure 4: Creating a practical lowpass filter

i. Find the time signals $h_1(t)$ and $h_2(t)$ that are the inverse Fourier Transforms of $H_1(f)$ and $H_2(f)$.
ii. What would \( h(t) \), the inverse Fourier Transform of \( H(f) \) be? (Hint: \( H(f) = H_1(f) \ast H_2(f) \), which is what in the time domain?)

iii. Why do you think \( H(f) \) is a more practical filter? Think about this question in terms of \( h(t) \).

3. Digital Modulation

(a) Describe the relationships between receiver noise, intersymbol interference (ISI), and bit rate error. How does each affect the constellation and eye diagrams?

(b) Repeat Problem 1 of HW#3 with an input of (-3,3). Do this by breaking the sinusoids into complex exponentials rather than the graphical method discussed in lecture.

4. Energy and Noise

(a) Considering the three PDFs in Figure 5. Order them in terms of their variances using your intuition. Do not perform detailed calculations.

![Figure 5: Different PDFs for noise](image)

(b) A digital transmitter sends out signals that have a high of 5V and a low of 0V. For some reason the channel has a strange noise distribution shown in Figure 6:

![Figure 6: Distribution for noise in channel](image)

i. What should the slicing threshold be set as?

ii. Specify the SNR in dB for the system.

iii. Determine the bit error rate of the system.