1. (a) If the input is $(-1,3)$, then
\[ y(t) = (-1)2 \cos(2\pi 10^8 t) + (3)2 \sin(2\pi 10^8 t) \]
\[ = -e^{j2\pi 10^8 t} - e^{-j2\pi 10^8 t} + \frac{3}{j}e^{j2\pi 10^8 t} - \frac{3}{j}e^{-j2\pi 10^8 t} \]
\[ = (-1 - 3j)e^{2\pi 10^8 t} + (-1 - 3j)e^{2\pi 10^8 t} \]
\[ = -\sqrt{10}e^{j\tan^{-1}(3)}e^{j2\pi 10^8 t} - \sqrt{10}e^{j\tan^{-1}(-3)}e^{j2\pi 10^8 t} \]
\[ = -2\sqrt{10} \cos(2\pi 10^8 t + \tan^{-1}(3)) \]
\[ = 2\sqrt{10} \cos(2\pi 10^8 t + \tan^{-1}(3) + \pi) \]
\[ \Rightarrow A = 2\sqrt{10} \]
\[ f = 10^8 \]
\[ \phi = \tan^{-1}(3) + \pi \]

Note that there are alternative ways of looking at this problem. One can solve it by using trig identities or considering a graphical method. Noting Figure 1, A is simply the double the distance from origin to the point on the I-Q plane. The phase $\phi$ is the CW angle from the line $I = 0$. However, it is a good idea to remember that we can always solve this problem by manipulating complex exponentials.

![Figure 1: Graphical representation of Question 1](image)

(b) If the input is $(3,-3)$, then
\[ y(t) = (3)2 \cos(2\pi 10^8 t) + (-3)2 \sin(2\pi 10^8 t) \]
\[ = 3e^{j2\pi 10^8 t}3e^{-j2\pi 10^8 t} - \frac{3}{j}e^{j2\pi 10^8 t} + \frac{3}{j}e^{-j2\pi 10^8 t} \]
\[ = (3 + 3j)e^{2\pi 10^8 t} + (3 - 3j)e^{2\pi 10^8 t} \]
\[ = \sqrt{18}e^{j\tan^{-1}(1)}e^{j2\pi 10^8 t} + \sqrt{18}e^{j\tan^{-1}(-1)}e^{j2\pi 10^8 t} \]
\[ = 2\sqrt{18} \cos(2\pi 10^8 t + \tan^{-1}(1)) \]
\[ A = 2\sqrt{18} \]
\[ f = 10^8 \]
\[ \phi = \tan^{-1}(1) \]

2. (a) For the I channel, we want to choose a slicing level exactly between the maximum value for logic 0 and the minimum value for a logic 1.
Slicing level for I = \( \frac{1+(-.5)}{2} = .25 \)

(b) The constellation diagram for the I should look something like:

![Figure 2: Constellation of I Channel](image)

(c) The Q channel slicing level should also be exactly between the maximum value for logic 0 and the minimum value for a logic 1.
Slicing level for I = \( \frac{1+.5}{2} = -.25 \)

(d) The constellation diagram for the Q should look something like:

![Figure 3: Constellation of Q Channel](image)
(e) The constellation diagram of the sum of I and Q is:

![Constellation Diagram](image)

Figure 4: Constellation of combined I and Q Channel

(f) Define the noise margin as distance from slicing level to bounds of the signal. Using the slicing levels of parts a and c, we can achieve a maximal noise margin of 0.75.

3. (a) Let \( x'(t) = e^{2\pi f_{\text{shift}} t} x(t) \)

\[
X'(f) = \int_{-\infty}^{\infty} x'(t) e^{-j2\pi ft} dt \\
= \int_{-\infty}^{\infty} x(t) e^{2\pi f_{\text{shift}} t} e^{-j2\pi ft} dt \\
= \int_{-\infty}^{\infty} x(t) e^{2\pi (f-f_{\text{shift}}) t} dt \\
= X(f-f_{\text{shift}})
\]

\[ \Rightarrow e^{j2\pi f_{\text{shift}} t} x(t) \Leftrightarrow X(f-f_{\text{shift}}) \]

(b) The signals \( X_1(t) \) and \( X_2(t) \) are given below:

![Spectrum Diagram](image)

Figure 5: Spectrum of \( X_1(t) \) and \( X_2(t) \)
(c) We want to shift the spectrum by 1 MHz to the left. Thus, $z(t) = e^{j2\pi(1\text{MHz})t}$.

The resulting $X_3(t)$ and $Y(t)$ are the following:

![Figure 6: Spectrum of $X_3(t)$ and $Y(t)$](image)

(d) If $r(t) = \cos(2\pi(10\text{MHz})t)$, no choice of $z(t)$ can recover the desired signal. This is because the two spectra will overlap in $x_1(t)$ and thus cannot be separated. Using this choice of $r(t)$, we can see $X_1(f)$ will look like

![Figure 7: Spectrum of $X_1(t)$](image)