abstract data types

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plan for today

topics

• datatypes and structure
• the idea of data abstraction
• types and operations for DPLL
• example abstract types & design challenges
• designing an equals operation

patterns

• Factory Method (in Literal)
what's wrong with our solver?
a missed opportunity

look at what happens

\cdot consider formula

\[ \text{Socrates} \implies \text{Human} \land \text{Human} \implies \text{Mortal} \land \neg (\text{Socrates} \implies \text{Mortal}) \]

\cdot suppose order or evaluation is \text{Socrates, Human, Mortal}

\cdot and suppose we set \text{Socrates} to true

\cdot then clearly must set \text{Human} to true

\cdot and then must set \text{Mortal} to true...

\cdot but our solver ignores all this!

next time

\cdot a real SAT solver

\cdot implements this scheme with unit propagation
the idea of abstract types
revisiting var set

recall computing set of vars appearing in a formula

• declare function
  \[ \text{vars: } F \rightarrow \text{Set<Var>} \]

• declare datatype
  \[ F = \text{Var(name: String)} + \text{Or(left:F, right:F)} + \text{And(left:F, right:F)} + \text{Not(formula:F)} \]

• define function over variants
  \[
  \begin{align*}
  \text{vars (Var(n))} &= \{\text{Var(n)}\} \\
  \text{vars (Or(fl, fr))} &= \text{vars(fl)} \cup \text{vars(fr)} \\
  \text{vars (And(fl, fr))} &= \text{vars(fl)} \cup \text{vars(fr)} \\
  \text{vars (Not(f))} &= \text{vars(f)}
  \end{align*}
  \]

where do sets come from?

• could define \textit{structurally} like this
  \[
  \text{Set<T>} = \text{List<T>}
  \]

• but better to define by \textit{operations} instead: \{\}, \cup
public interface Set<E> {
    public Set<E> add (E e);
    public Set<E> remove (E e);
    public Set<E> addAll (Set<E> s);
    public boolean contains (E e);
    public E choose ()
    public boolean isEmpty ();
    public int size ();
}

a set interface
a set implementation

```java
public class ListSet<E> implements Set<E> {
    private List<E> elements;

    public ListSet () {elements = new EmptyList<E> ();} 

    public Set<E> add (E e) {
        if (elements.contains (e)) return this;
        return new ListSet<E> (elements.add (e));
    }

    public Set<E> remove (E e) {
        if (isEmpty()) return this;
        E first = elements.first();
        ListSet<E> rest = new ListSet<E> (elements.rest());
        if (first.equals(e))
            return rest;
        else
            return rest.remove(e).add(first);
    }

    public boolean contains (E e) {
        return elements.contains(e);
    }

    ...
```
a new viewpoint

datatype productions
\cdot datatypes defined by their structure or representation

abstract datatypes
\cdot datatypes defined by their operations or behavior

extending the type repertoire
\cdot used to thinking of basic types behaviorally:
  
  integers: +, *, <, =
  
  array: get(a, i), store(a, i, e)

\cdot abstract datatypes: user-defined types
  
  string: concat(s, t), charAt(s, i)
  
  set: {}, \cup, \in
what makes an abstract type?

defined by operations
· an integer is something you can add, multiply, etc
· a set is something you can test membership in, union, etc

representation is hidden or “encapsulated”
· client can’t see how the type is represented in memory
· is integer twos-complement? big or little endian?
· is set a list? a binary tree? an array?

language support for data abstraction
· packaging operations with representations
· hiding representation from clients
encapsulation

two reasons for encapsulation of representations

rep independence

• if client can't see choice of rep, implementor can change it
• eg: integers: your program can run on a different platform
• eg: sets: programmer can switch rep from list to array

rep invariants

• not all values of the rep make legal abstract values
• prevent client from accessing rep so code of ADT can preserve invariants
• eg: sets: make sure element does not appear twice
classic types

domain specific and generic types

- some types are specific to a domain (clause, literal)
- some have wide application (list, set)
- widely applicable types are usually polymorphic
- these are the “classic ADTs”

in Java

- found in the standard package java.util
- often called “Java collection framework”
## a zoo of types

<table>
<thead>
<tr>
<th>type</th>
<th>overview</th>
<th>producers</th>
<th>observers</th>
<th>common reps</th>
</tr>
</thead>
<tbody>
<tr>
<td>list</td>
<td>sequence for concatenation and front-append</td>
<td>add, append</td>
<td>first, rest, ith</td>
<td>array, linked list</td>
</tr>
<tr>
<td>queue</td>
<td>FIFO: first in, first out</td>
<td>enq, deq</td>
<td>first</td>
<td>array, list, circular buffer</td>
</tr>
<tr>
<td>stack</td>
<td>LIFO: last in, first out</td>
<td>push, pop</td>
<td>top</td>
<td>array, list</td>
</tr>
<tr>
<td>map</td>
<td>associates keys and values</td>
<td>put</td>
<td>get</td>
<td>association list, hash table, tree</td>
</tr>
<tr>
<td>set</td>
<td>unordered collection</td>
<td>insert, remove</td>
<td>contains</td>
<td>map, list, array, bitvector, tree</td>
</tr>
<tr>
<td>bag</td>
<td>like set, but element can appear more than once</td>
<td>insert, remove</td>
<td>count</td>
<td>map, array, association list</td>
</tr>
</tbody>
</table>

**note**

- producers and observers: just examples
- common reps: some (eg, hash table, bitvector) just for mutable versions
the DPLL algorithm
what types do you need?

a square root procedure needs
  ‣ floating point numbers

a SAT solver needs
  ‣ booleans, literals, clauses, environments

characteristic of complex programs
  ‣ computations defined over set of datatypes
  ‣ most of the datatypes are not built-in, but user-defined
  ‣ so design datatypes before other program components

let’s examine the DPLL algorithm
  ‣ and see what types it needs
basic backtracking algorithm

clausal form
• recall that algorithm acts on formula represented as clause-set
• product of sums: need every clause true, some literal in each clause

elements of the algorithm
• backtracking search: pick a literal, try false then true
• if clause set is empty, success
• if clause set contains empty clause, failure

element
• want to prove Socrates⇒Mortal from Socrates⇒Human ∧ Human⇒Mortal
• so give solver: Socrates⇒Human ∧ Human⇒Mortal ∧ ¬ (Socrates⇒Mortal)
• in clausal form: {{¬Socrates,Human},{¬Human,Mortal},{Socrates},{¬Mortal}}
• in shorthand: {SH}{HM}{S}{M}
backtracking execution

\{SH\}\{HM\}\{S\}\{M\}

\set H

\{M\}\{S\}\{M\} \quad \{S\}\{S\}\{M\}

\set H

\set M

\{S\}\{} \quad \{}\{S\} \quad \{S\}\{S\}\{} \quad \{S\}\{S\}

\set M

\set M

\set S

\set S

\{} \quad \{}

\quad \set M

\quad \set M

\quad \set S

\quad \set S

\cdot \text{stop when node contains } \{} (\text{failure}) \text{ or is empty (success)}

\cdot \text{in this case, all paths fail, so theorem is valid}

\cdot \text{in worst case, number of leaves is } 2^{\#\text{liters}}
DPLL

classic SAT algorithm
\· Davis-Putnam-Logemann-Loveland, 1962

unit propagation
\· on top of backtracking search
\· if a clause contains one literal, set that literal to true

example (on right)
\· in this case, no splitting needed
\· propagate S, then H, then M

performance
\· often much better, but worst case still exponential
an implementation

```java
public static Environment solve(List<Clause> clauses) {
    return solve(clauses, new Environment());
}

private static Environment solve(List<Clause> clauses, Environment env) {
    if (clauses.isEmpty()) return env; // if no clauses, trivially solvable
    Clause min = null;
    for (Clause c : clauses) {
        if (c.isEmpty()) return null; // if empty clause found, then unsat
        if (min == null || c.size() < min.size()) min = c;
    }
    Literal l = min.chooseLiteral();
    bool.Variable v = l.getVariable();
    if (min.isUnit()) { // a unit clause was found, so propagate
        env = env.put(v, l instanceof PosLiteral ? Bool.TRUE : Bool.FALSE);
        return solve(reduceClauses(clauses, l), env);
    } // else split
    if (l instanceof NegLiteral) l = l.getNegation();
    Environment solvePos = solve(reduceClauses(clauses, l), env.put(v, Bool.TRUE));
    if (solvePos == null)
        return solve(reduceClauses(clauses, l.getNegation()), env.put(v, Bool.FALSE));
    else return solvePos;
}

private static List<Clause> reduceClauses(List<Clause> clauses, Literal l) {
    List<Clause> reducedClauses = new EmptyList<Clause>();
    for (Clause c : clauses) {
        Clause r = c.reduce(l);
        if (r != null)
            reducedClauses = reducedClauses.add(r);
    }
    return reducedClauses;
}
```
basic types for SAT
public static Environment solve(List<Clause> clauses) {
    return solve(clauses, new Environment());
}

private static Environment solve(List<Clause> clauses, Environment env) {
    if (clauses.isEmpty()) return env; // if no clauses, trivially solvable
    Clause min = null;
    for (Clause c : clauses) {
        if (c.isEmpty()) return null; // if empty clause found, then unsat
        if (min == null || c.size() < min.size()) min = c;
    }
    Literal l = min.chooseLiteral();
    bool.Variable v = l.getVariable();
    if (min.isUnit()) { // a unit clause was found, so propagate
        env = env.put(v, l instanceof PosLiteral ? Bool.TRUE : Bool.FALSE);
        return solve(reduceClauses(clauses, l), env);
    } // else split
    if (l instanceof NegLiteral) l = l.getNegation();
    Environment solvePos = solve(reduceClauses(clauses, l), env.put(v, Bool.TRUE));
    if (solvePos == null)
        return solve(reduceClauses(clauses, l.getNegation()), env.put(v, Bool.FALSE));
    else return solvePos;
}

private static List<Clause> reduceClauses(List<Clause> clauses, Literal l) {
    List<Clause> reducedClauses = new EmptyList<Clause>();
    for (Clause c : clauses) {
        Clause r = c.reduce(l);
        if (r != null)
            reducedClauses = reducedClauses.add(r);
    }
    return reducedClauses;
}
introduced my own boolean ADT

- has three boolean values: TRUE, FALSE and UNDEFINED
- why did I do this?

```java
public enum Bool {
    TRUE, FALSE, UNDEFINED;

    public Bool and (Bool b) {
        if (this==FALSE || b==FALSE) return FALSE;
        if (this==TRUE && b==TRUE) return TRUE;
        return UNDEFINED;
    }

    public Bool or (Bool b) {
        if (this==FALSE && b==FALSE) return FALSE;
        if (this==TRUE || b==TRUE) return TRUE;
        return UNDEFINED;
    }

    public Bool not () {
        if (this==FALSE) return TRUE;
        if (this==TRUE) return FALSE;
        return UNDEFINED;
    }
}
```
should **Environment** be an ADT at all?

• just a mapping from literals to booleans
• decided yes, in case I wanted to add functionality later
• sure enough, I did: return `Bool.UNDEFINED` if no mapping

```java
public class Environment {
    private Map<Variable, Bool> bindings;

    public Environment put(Variable v, Bool b) {
        return new Environment (bindings.put (v, b));
    }

    public Bool get(Variable v){
        Bool b = bindings.get(v);
        if (b==null) return Bool.UNDEFINED;
        else return b;
    }

    ...
}
```
clause type

what's a clause?

- clause is disjunction of set of literals; empty means FALSE, no rep of TRUE

```java
public class Clause {
    public Clause() {...}
    public Clause(Literal literal) {...}
    public Clause add(Literal l) {...}
    public Clause reduce(Literal literal) {...}
    public Literal chooseLiteral() {...}
    public boolean isUnit() {...}
    public boolean isEmpty() {...}
    public int size() {...}
}
```

notes

- order not exposed in observers: chooseLiteral is non-deterministic
- isUnit, isEmpty are for convenience of clients, not strictly necessary
- add, reduce are the key ‘producers’:
  - add (l): return clause obtained by adding l as a disjunct
  - reduce (l): return clause obtained by setting l to TRUE
designing operations

issue

‣ what should add, reduce return when result is \texttt{TRUE}? eg, add \texttt{S} to \{\texttt{S}\}

design options

‣ create clause for special value \texttt{TRUE}
‣ throw an exception
‣ return \texttt{null}

considerations

‣ clause set should not contain vacuous \texttt{TRUE} clauses
‣ exceptions are awkward; in Java, best used only when not expected
‣ compiler doesn’t ensure that null return value is checked
what's wrong with this code?

\`standard recursive pattern applied to reduce, but wasteful\`

```java
/**
 * Requires: literal is non-null
 * @return clause obtained by setting literal to true
 * or null if the entire clause becomes true
 */
public Clause reduce(Literal literal) {
    List<Literal> reducedLiterals = reduce(literals, literal);
    if (reducedLiterals == null) return null;
    else return new Clause(reducedLiterals);
}

private static List<Literal> reduce(List<Literal> literals, Literal l) {
    if (literals.isEmpty()) return literals;
    Literal first = literals.first();
    List<Literal> rest = literals.rest();
    if (first.equals(l)) return null;
    else if (first.equals(l.getNegation())) return rest;
    else {
        List<Literal> restR = reduce(rest, l);
        if (restR == null) return null;
        return restR.add(first);
    }
```
private static List<Literal> reduce(List<Literal> literals, Literal l) {
    if (literals.isEmpty()) return literals;
    Literal first = literals.first();
    List<Literal> rest = literals.rest();
    if (first.equals(l)) return null;
    else if (first.equals(l.getNegation())) return rest;
    else {
        List<Literal> restR = reduce(rest, l);
        if (restR == null) return null;
        if (restR == rest) // then clause is unchanged beyond this point
            return literals;
        else return restR.add(first);
    }
}

idea

• avoiding rebuilding the list except when needed
• may save a lot of allocation and garbage collection
representation independence
choice of rep

an abstract type can be implemented with different reps

' example: two versions of Environment

```java
public class Environment {
    private Map<Variable, Bool> bindings;
    ...
    public Bool get(Variable v){
        Bool b = bindings.get(v);
        if (b==null) return Bool.UNDEFINED;
        else return b;
    }
}

public class Environment {
    private Set<Variable> trues, falses;
    ...
    public Bool get(Variable v){
        if (trues.contains (v)) return Bool.TRUE;
        if (falses.contains (v)) return Bool.FALSE;
        return Bool.UNDEFINED;
    }
}
```
achieving rep independence

rep independence

\· want to be able to change rep without changing client

what does this require?

\· if client can access fields directly
  \> rep is fully “exposed”: heavy modification of client code required
\· if client calls methods that return fields directly
  \> can fix by modifying ADT methods, but will be ugly
\· if client can’t access fields even indirectly (as in previous slide)
  \> ADT is easily modified locally

so independence is achieved by

\· combination of language support and programmer discipline
designing equality
comparing literals

need to compare literals

- eg, in Clause.reduce
  - eg, when $S$ is true: \{SH\} reduces to \{H\}, and \{SH\} reduces to $\text{TRUE}$

- a SAT solver will do this a lot, so must be efficient

equality of immutable types

- calling constructor twice on same args gives distinct objects
  - \text{Literal} a = \text{new Literal} ("S");
  - \text{Literal} b = \text{new Literal} ("S");
  - \text{System.out.println} (a==b ? "same" : "not"); \hspace{1em} // prints not

two strategies

- use \text{equals} method, and code it to compare object values
  - for literals, compare names char-by-char every time!

- \text{intern} the objects so there's at most one object with a given value
interning with a factory method

factory method pattern

• instead of constructor, client calls a static ‘factory’ method

```java
public static T make () { return new T(); }
```

• factory method can call constructor, but can also recycle objects

```java
public abstract class Literal {
    protected Literal negation;
    protected Variable var;
    public Literal (Variable name) {this.var = new bool.Variable(name);}
}
public class Pos extends Literal {
    protected static Map<String,Pos> alloc = new ListMap<String,Pos>();
    private Pos (String name) {super(name);}
    public static Pos make (String name) {
        Pos l = alloc.get(name);
        if (l==null) {
            l = new Pos(name);
            Neg n = new Neg(name);
            l.negation = n; n.negation = l;
            alloc = alloc.put(name, l);
        }
        return l;
    }
}
```
putting it all together: demo
allocating variables

Sudoku abstract type contains

- 2D array of known values (square)
- 3D array of boolean variables (occupies)

```java
public class Sudoku {
    private final int dim;
    private final int size;
    private int[][] square;
    private Formula[][][] occupies;

    public Sudoku (int dim) {
        this.dim = dim;
        size = dim * dim;
        square = new int[size][size];
        occupies = new Formula[size][size][size];
        for (int i = 0; i < size; i++)
            for (int j = 0; j < size; j++)
                for (int k = 0; k < size; k++) {
                    Formula l = Formula.makeVariable("occupies(" + i + "," + j + "," + k + ")");
                    occupies[i][j][k] = l;
                }
    }

    public static Sudoku fromFile (String filename, int dim) {...}
```
to create formula

• create at-most and at-least formulas per row, column, block
• my solver converts to CNF

```java
public Formula getFormula () {
    Formula formula = Formula.TRUE;
    // each symbol appears exactly once in each row
    for (int k = 0; k < size; k++) {
        for (int i = 0; i < size; i++) {
            Formula atMost = Formula.TRUE;
            Formula atLeast = Formula.FALSE;
            for (int j = 0; j < size; j++) {
                atLeast = atLeast.or (occupies[i][j][k]);
                for (int j2 = 0; j2 < size; j2++)
                    if (j != j2)
                        atMost = atMost.and (occupies[i][j][k].implies(
                                            occupies[i][j2][k].not()));
            }
            formula = formula.and (atMost).and (atLeast);
        }
    }
    return formula;
}
```
interpreting the solution

to interpret solution

' just iterate over puzzle, and look up each variable in environment

```java
public String interpretSolution (Environment e) {
    String result = "";
    for (int i = 0; i < size; i++) {
        String row = "|";
        for (int j = 0; j < size; j++)
            for (int k = 0; k < size; k++) {
                Formula l = occupies[i][j][k];
                if (l.eval(e) == Bool.TRUE)
                    row = row + (k+1) + "|";
            }
        result = result + row + "\n";
    }
    return result;
}
```
executing the solver

steps

• create Sudoku object from file

• extract formula, solve and interpret

```java
public static void solveStandardPuzzle (String filename) throws IOException {
    long started = System.nanoTime();
    System.out.println("Parsing...");
    Sudoku s = Sudoku.fromFile (filename, 3);
    System.out.println("Creating SAT formula...");
    Formula f = s.getFormula();
    System.out.println("Solving...");
    Environment e = f.solve();
    System.out.println("Interpreting solution...");
    String solution = s.interpretSolution(e);
    System.out.println("Solution is: \n" + solution);
    long time = System.nanoTime();
    long timeTaken = (time - started);
    System.out.println("Time:" + timeTaken/1000000 + "ms");
}
```
sample run

solving a sample Sudoku puzzle

• 1,000 variables and 24,000 clauses
• about 6 seconds (on 2.4GHz Intel Mac with 2GB memory)

Parsing...
Creating SAT formula...
Solving...
Interpreting solution...
Solution is:
|9|1|6|8|4|3|5|2|7|
|8|4|2|7|5|6|9|3|1|
|7|5|3|2|9|1|8|6|4|
|3|6|4|9|2|7|1|8|5|
|2|8|1|5|6|4|7|9|3|
|1|9|7|1|3|8|2|4|6|
|6|7|8|4|1|9|3|5|2|
|4|2|9|3|7|5|6|1|8|
|1|3|5|6|8|2|4|7|9|

Time: 6021ms
features of modern SAT solvers
modern SAT solvers

some great open-source SAT solvers

• Sat4J (all Java) http://www.sat4j.org/
• Chaff http://www.princeton.edu/~chaff
• Berkmin http://eigold.tripod.com/BerkMin.html
• MiniSat http://minisat.se/

what do they do beyond what I’ve explained?

• learning: if literal choices ABC ended in failure, add \{ABC\}
• splitting heuristics: pick the literal to split on carefully
• randomization: restart with new literal order
• clever representation invariants (explained later in course)

a less conventional SAT solver

summary
summary

principles
\cdot define an abstract type by its operations
\cdot hide the representation from clients

patterns
\cdot Factory Method

design issues
\cdot collapsing clauses
\cdot optimizing reduce