Sampling, Aliasing, & Mipmaps
Examples of Aliasing

Original Image  Samples  Reconstruction

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Examples of Aliasing

Jagged boundaries
Examples of Aliasing

Improperly rendered detail
Examples of Aliasing

Texture Errors
In photos too
Philosophical perspective

- The physical world is continuous, inside the computer things need to be discrete
- Lots of computer graphics is about translating continuous problems into discrete solutions
  - e.g. ODEs for physically-based animation, global illumination, meshes to represent smooth surfaces, rasterization, antialiasing
- Careful mathematical understanding helps do the right thing
What is a Pixel?

• A pixel is not:
  – a box
  – a disk
  – a teeny tiny little light

• A pixel “looks different” on different display devices

• A pixel is a sample
  – it has no dimension
  – it occupies no area
  – it cannot be seen
  – it has a coordinate
  – it has a value
More on Samples

• In signal processing, the process of mapping a continuous function to a discrete one is called *sampling*.

• The process of mapping a continuous variable to a discrete one is called *quantization*.
  – Gamma helps quantization.

• To represent or render an image using a computer, we must both sample and quantize.
  – Today we focus on the effects of sampling and how to fight them.

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Sampling & reconstruction

The visual array of light is a continuous function

1/ we sample it
   – with a digital camera, or with our ray tracer
   – This gives us a finite set of numbers, not really something we can see
   – We are now inside the discrete computer world

2/ we need to get this back to the physical world:
   we reconstruct a continuous function
   – for example, the point spread of a pixel on a CRT or LCD

• Both steps can create problems
  – pre-aliasing caused by sampling
  – post-aliasing caused by reconstruction
  – We focus on the former
Questions?
Sampling Density

- If we’re lucky, sampling density is enough
Sampling Density

- If we insufficiently sample the signal, it may be mistaken for something simpler during reconstruction (that's aliasing!)
- This is why it’s called aliasing: the new low-frequency sine wave is an alias/ghost of the high-frequency one
Discussion

• Types of aliasing
  – Edges
    • mostly directional aliasing (vertical and horizontal edges rather than actual slope)
  – Repetitive textures
    • Paradigm of aliasing
    • Harder to solve right
    • Motivates fun mathematics
Solution?

• How do we avoid that high-frequency patterns mess up our image?

• We blur!
  – In the case of audio, people first include an analog low-pass filter before sampling
  – For ray tracing/rasterization: compute at higher resolution, blur, resample at lower resolution
  – For textures, we can also blur the texture image before doing the lookup

• To understand what really happens, we need serious math
Types of blur

• Blur = local weighted average
• Different set of weights (kernel) => different blur
  – e.g.: box, Gaussian, bilinear, bicubic
• Yields slightly different quality’
• Will discuss later
Questions?
Supersampling in graphics

- Pre-filtering (blurring before sampling) is hard
  - Requires analytical visibility
  - Then difficult to integrate analytically with filter
- Possible for lines, or if visibility is ignored
- Usually, fall back to supersampling
In practice: Supersampling

- Your intuitive solution is to compute multiple color values per pixel and average them.

jaggies w/ antialiasing
Uniform supersampling

- Compute image at resolution $k \times \text{width}$, $k \times \text{height}$
- Downsample using low-pass filter (e.g. Gaussian, sinc, bicubic)
Different kinds of blur (/average)

From the PBRT book

Figure 7.37: The pixel reconstruction filter used to convert the image samples into pixel values can have a noticeable effect on the character of the final image. Here we see blowups of a region of the brick wall in the Sponza atrium scene, filtered with (a) the box filter, (b) Gaussian, and (c) Mitchell-Netravali filter. Note that the Mitchell filter gives the sharpest image, while the Gaussian blurs it. The box is the least desirable, since it allows high-frequency aliasing to leak into the final image. (Note artifacts on the top edges of arches, for example.)
In practice: Supersampling

• Your intuitive solution is to compute multiple color values per pixel and average them

• A better interpretation of the same idea is that
  – You first create a higher resolution image
  – You blur it (low pass, prefilter)
  – You resample it at a lower resolution
Uniform supersampling

• Advantage:
  – The first (super)sampling captures more high frequencies that are not aliased
  – Downsampling can use a good filter

• Issues
  – Frequencies above the (super)sampling limit are still aliased

• Works well for edges, since spectrum replication is less an issue
• Not as well for repetitive textures
  – But solution soon

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Questions?
Uniform supersampling

- Problem: supersampling only pushes the problem further: The signal is still not bandlimited
- Aliasing happens
Jittering

- Uniform sample + random perturbation
- Sampling is now non-uniform
- Signal processing gets more complex
- In practice, adds noise to image
- But noise is better than aliasing Moiré patterns
Figure 11
Jittered sampling of a slowly moving texture with jitter of 0, .5, and 1 from left to right and oversampling rates of 1 and 2 from top to bottom.
Jittering

- Displaced by a vector a fraction of the size of the subpixel distance
- Low-frequency Moire (aliasing) pattern replaced by noise
- Extremely effective
- Patented by Pixar!
- When jittering amount is 1, equivalent to stratified sampling (cf. later)
Questions?
Sampling Texture Maps

- How to map the texture area seen through the pixel window to a single pixel value?
Sampling Texture Maps

- When texture mapping it is rare that the screen-space sampling density matches the sampling density of the texture.

Original Texture

64x64 pixels

Magnification for Display

Minification for Display

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Linear Interpolation

• Tell OpenGL to use a tent filter instead of a box filter.
• Magnification looks better, but blurry
  – (texture is under-sampled for this resolution)
  – Oh well.
Questions?
Minification: Examples of Aliasing
Solution

• 1/ we can supersample the final image
  – But only offsets the Nyquist limit

• 2/ we can “blur” the texture (prefilter)
  – The right way to go

But how much blur?
Spatial Filtering

- Remove the high frequencies which cause artifacts in texture minification.
- Compute a spatial integration over the extent of the pixel.
- This is equivalent to convolving the texture with a filter kernel centered at the sample (i.e., pixel center)!
- Expensive to do during rasterization, but an approximation it can be precomputed.
MIP Mapping

- Construct a pyramid of images that are pre-filtered and re-sampled at $1/2$, $1/4$, $1/8$, etc., of the original image's sampling.
- During rasterization we compute the index of the decimated image that is sampled at a rate closest to the density of our desired sampling rate.
- MIP stands for *multum in parvo* which means *many in a small place*.
MIP Mapping Example

Nearest Neighbor

MIP Mapped (Bi-Linear)
MIP Mapping Example

- Small details may "pop" in and out of view

Nearest Neighbor

MIP Mapped (Bi-Linear)
Examples of Aliasing

Texture Errors

nearest neighbor/point sampling

mipmaps & linear interpolation

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Finding the mip level

- Square MIP-map area is a bad approximation
Finding the MIP level

How does a screen-space change $dt$ relates to a texture-space change $du, dv$.

$=>$ derivatives, $(du/dt, dv/dt)$.

e.g. computed by hardware during rasterization

often: finite difference (pixels are handled by quads)
Actually, you have a choice of ways to translate this derivative value into a MIP level.

Because we have two derivatives, for u and for v (anisotropy)

This also brings up one of the shortcomings of MIP mapping. MIP mapping assumes that both the u and v components of the texture index are undergoing a uniform scaling, while in fact the terms $du/dt$ and $dv/dt$ are relatively independent. Thus, we must make some sort of compromise. Two of the most common approaches are given below:

\[
\text{level} = \log_2 \left( \sqrt{ \left( \frac{du}{dt} \right)^2 + \left( \frac{dv}{dt} \right)^2 } \right)
\]

\[
\text{level} = \log_2 \left( \max \left( \left| \frac{du}{dt} \right|, \left| \frac{dv}{dt} \right| \right) \right)
\]
Anisotropy & MIP-Mapping

• What happens when the surface is tilted?

Nearest Neighbor  MIP Mapped (Bi-Linear)

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Questions?
Elliptical weighted average

- Isotropic filter wrt screen space
- Becomes anisotropic in texture space
- e.g. use anisotropic Gaussian
- Called Elliptical Weighted Average (EWA)

Figure 3: A perspective projection of a Gaussian filter into texture space

Figure 4: An affine projection of a Gaussian filter into texture space.
Image Quality Comparison

- Trilinear mipmapping

EWA  trilinear mipmapping

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Approximation of anisotropic

- Feline: Fast Elliptical Lines for Anisotropic Texture Mapping Joel McCormack, Ronald Perry, Keith I. Farkas, and Norman P. Jouppi SIGGRAPH 1999


- Approximate Anisotropic Gaussian by a set of isotropic “probes”

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Figure 10: Trilinear paints curved lines with blurring.

Figure 13: “High-quality” Simple Feline paints curved lines with few artifacts.

Figure 14: Mip-mapped EWA paints curved lines with few artifacts.
Signal processing 101

- Sampling and filtering are best understood in terms of Fourier analysis
- We already saw that sine waves generate aliasing: a high frequency sine wave turns into a low frequency one when undersampled
Blurring: convolution

Input

Kernel

Convolution sign

Output

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Blurring: convolution

Same shape, just reduced contrast!!!

This is an eigenvector
(output is the input multiplied by a constant)
Motivation for Fourier analysis

- Sampling/reconstruction is a linear process
- The sampling grid is a periodic structure
  - Fourier is pretty good at handling periodic stuff
- Plus we saw that a sine wave has serious problems with sampling
- The solution is about blurring
  - We have seen that sine wave are simple wrt blur
- In general, the Fourier transform is just a change of basis
  - In that basis, aliasing & blurring are easier to understand
  - become diagonal (we can consider frequencies individually)
Questions?