Decision Analysis & Decision Support
6.872/HST.950

Tasks?

- Mechanics
  - Record keeping
  - Administration
  - Scheduling
  - ...
- Diagnosis
- Prognosis
- Therapy

Types of Decision Support

- “Doctor's Assistant” for clinicians at any level of training
- Expert (specialist) consultation for non-specialists
- Monitoring and error detection
- Critiquing, what-if
- Guiding patient-controlled care
- Education and Training
- Contribution to medical research
- ...

Two Historical Views on How to Build Expert Systems

- Great cleverness
  - Powerful inference abilities
  - Ab initio reasoning

- Great stores of knowledge
  - Possibly limited ability to infer, but
  - Vast storehouse of relevant knowledge, indexed in an easy-to-apply form
Change over 30 years

• 1970’s: human knowledge, not much data
• 2000's: vast amounts of data, traditional human knowledge (somewhat) in doubt
• Could we “re-discover” all of medicine from data? I think not!
• Should we focus on methods for reasoning with uncertain data? Absolutely!

Cancer Test

• We discover a cheap, 95% accurate test for cancer.
• Give it to “Mrs. Jones”, the next person who walks by 77 Mass Ave.
• Result is positive.
• What is the probability that Mrs. Jones has cancer?

Figuring out Cancer Probability

Assume Ca in 1% of general population:

\[ \begin{align*}
100,000 & \quad 95\% \\
99,000 & \quad + \quad 4,950 \\
950 & \quad + \quad 950 \\
950 + 4950 & = .161
\end{align*} \]

At the Extremes

• If Ca probability in population is 0.1%,
  – Then post positive result, \( p(Ca) = 1.87\% \)

• If Ca probability in population is 50%,
  – Then post-positive result, \( p(Ca) = 95\% \)
Bayes’ Rule

\[ P(D | T) = \frac{P(D)P(T | D)}{P(D)P(T | D) + P(D)P(T | \overline{D})} \]

Odds/Likelihood Form

\[ P(D | T) = \frac{P(D)P(T | D)}{P(D)P(T | D) + P(\overline{D})P(T | \overline{D})} \]
\[ P(\overline{D} | T) = \frac{P(\overline{D})P(T | \overline{D})}{P(D)P(T | D) + P(\overline{D})P(T | \overline{D})} \]
\[ \frac{P(D | T)}{P(\overline{D} | T)} = \frac{P(D)}{P(\overline{D})} \cdot \frac{P(T | D)}{P(T | \overline{D})} \]
\[ O(D | T) = O(D)L(T | D) \]
\[ W(D | T) = W(D) + W(T | D) \]

DeDombal, et al. Experience 1970’s & 80’s

• “Idiot Bayes” for appendicitis
• 1. Based on expert estimates -- lousy
• 2. Statistics -- better than docs
• 3. Different hospital -- lousy again
• 4. Retrained on local statistics -- good

Rationality

• Behavior is a continued sequence of choices, interspersed by the world’s responses
• Best action is to make the choice with the greatest expected value
• … decision analysis
Example: Gangrene

- From Pauker’s “Decision Analysis Service” at New England Medical Center Hospital, late 1970's.
- Man with gangrene of foot
- Choose to amputate foot or treat medically
- If medical treatment fails, patient may die or may have to amputate whole leg.
- What to do? How to reason about it?

Decision Tree for Gangrene

Evaluating the Decision Tree

Decision Analysis: Evaluating Decision Trees

- Outcome: directly estimate value
- Decision: value is that of the choice with the greatest expected value
- Chance: expected value is sum of (probabilities x values of results)
- "Fold back" from outcomes to current decision.
- Sensitivity analyses often more important than result(!)
HELP System uses D.A.

Utility Analysis of Appendectomy

PROB OF APPENDICITIS

A APPENDICITIS BY HISTORY
B REBOUND TENDERNESS IN RLQ
C PRIOR APPENDECTOMY
D IF C THEN EXIT
E WHITE BLOOD COUNT (WBCx100) TH/M3, LAST
F PROB B A 620 90
G PROB F 43 18 9, 74 23 7, 93 18 11, 108 10 11, 121 16 13, 134 6 16, 151 5 16, 176 4 14
FVAL G

UTILITY OF APPENDECTOMY IS ESTIMATED AS $----
“Paint the Blackboards!”

DECISION | PATIENT STATE | UTILITY
---|---|---
Disease (p) | Treat disease

| | No disease (1-p) | Treat no disease

| | No treat | No treat disease

| | No disease (1-p) | No treat no disease

Test/Treat Threshold

Pauker, Kassirer, NEJM 1980

Threshold

- Benefit $B = U(\text{treat dis}) - U(\text{no treat dis})$
- Cost $C = U(\text{no treat no dis}) - U(\text{treat no dis})$
- Threshold probability for treatment:

$$T = \frac{1}{\frac{B}{C} + 1}$$

Pauker, Kassirer, NEJM 1975

Visualizing Thresholds
More Complex Decision Analysis Issues

- Repeated decisions
- Accumulating disutilities
- Dependence on history
- Cohorts & state transition models
- Explicit models of time
- Uncertainty in the uncertainties
- Determining utilities
  - Lotteries, …
- Qualitative models

Example: Acute Renal Failure

- Choice of a handful (8) of therapies (antibiotics, steroids, surgery, etc.)
- Choice of a handful (3) of invasive tests (biopsies, IVP, etc.)
- Choice of 27 diagnostic “questions” (patient characteristics, history, lab values, etc.)
- Underlying cause is one of 14 diseases
  - We assume one and only one disease

Decision Tree for ARF

- Choose:
  - Surgery for obstruction
  - Treat with antibiotics
  - Perform pyelogram
  - Perform arteriography
  - Measure patient’s temperature
  - Determine if there is proteinuria
  - …

Decision Tree for ARF

- Surgery for obstruction
  - Value = ???
- Treat with antibiotics
- Perform pyelogram
- Perform arteriography
  - Measure patient’s temperature
  - Determine if there is proteinuria
What happens when we act?

- Treatment: leads to few possible outcomes
  - different outcomes have different probabilities
    - probabilities depend on distribution of disease probabilities
  - value of outcome can be directly determined
    - value may depend on how we got there (see below)
    - therefore, value of a treatment can be determined by expectation

- Test: lead to few results, revise probability distribution of diseases, and impose disutility

- Questions: lead to few results, revise probability distribution

Initial probability distribution

<table>
<thead>
<tr>
<th>Disease</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATN Acute tubular necrosis</td>
<td>0.250</td>
</tr>
<tr>
<td>FARF Functional acute renal failure</td>
<td>0.400</td>
</tr>
<tr>
<td>OBSTR Urinary tract obstruction</td>
<td>0.100</td>
</tr>
<tr>
<td>AGN Acute glomerulonephritis</td>
<td>0.100</td>
</tr>
<tr>
<td>CN Renal cortical necrosis</td>
<td>0.020</td>
</tr>
<tr>
<td>HS Hepatorenal syndrome</td>
<td>0.005</td>
</tr>
<tr>
<td>PYE Pyelonephritis</td>
<td>0.010</td>
</tr>
<tr>
<td>AE Atheromatous Emboli</td>
<td>0.003</td>
</tr>
<tr>
<td>RI Renal infarction (bilateral)</td>
<td>0.002</td>
</tr>
<tr>
<td>RVT Renal vein thrombosis</td>
<td>0.002</td>
</tr>
<tr>
<td>VASC Renal vasculitis</td>
<td>0.050</td>
</tr>
<tr>
<td>SCL Scleroderma</td>
<td>0.002</td>
</tr>
<tr>
<td>CGAE Chronic glomerulonephritis, acute exacerbation</td>
<td>0.030</td>
</tr>
<tr>
<td>MH Malignant hypertension &amp; nephrosclerosis</td>
<td>0.030</td>
</tr>
</tbody>
</table>

ARF's Database: P(obs|D)

<table>
<thead>
<tr>
<th>Disease</th>
<th>Proteinuria 0</th>
<th>Proteinuria 2+</th>
<th>Proteinuria 3+ to 4+</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATN Acute tubular necrosis</td>
<td>0.1</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>FARF Functional acute renal failure</td>
<td>0.8</td>
<td>0.2</td>
<td>0.001</td>
</tr>
<tr>
<td>OBSTR Urinary tract obstruction</td>
<td>0.7</td>
<td>0.3</td>
<td>0.001</td>
</tr>
<tr>
<td>AGN Acute glomerulonephritis</td>
<td>0.01</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>CN Renal cortical necrosis</td>
<td>0.01</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>HS Hepatorenal syndrome</td>
<td>0.8</td>
<td>0.2</td>
<td>0.001</td>
</tr>
<tr>
<td>PYE Pyelonephritis</td>
<td>0.4</td>
<td>0.6</td>
<td>0.001</td>
</tr>
<tr>
<td>AE Atheromatous Emboli</td>
<td>0.1</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>RI Renal infarction (bilateral)</td>
<td>0.1</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>RVT Renal vein thrombosis</td>
<td>0.001</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>VASC Renal vasculitis</td>
<td>0.01</td>
<td>0.2</td>
<td>0.8</td>
</tr>
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<td>0.4</td>
<td>0.5</td>
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<td>0.2</td>
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<tr>
<td>MH Malignant hypertension &amp; nephrosclerosis</td>
<td>0.001</td>
<td>0.4</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Questions

- Blood pressure at onset
- proteinuria
- casts in urine sediment
- hematuria
- history of prolonged hypotension
- urine specific gravity
- large fluid loss preceding onset
- kidney size
- urine sodium
- strep infection within three weeks
- urine volume
- recent surgery or trauma
- age
- papilledema
- flank pain

- history of proteinuria
- symptoms of bladder obstruction
- exposure to nephrotoxic drugs
- disturbance in clotting mechanism
- pyuria
- bacteriuria
- sex
- transfusion within one day
- jaundice or ascites
- ischemia of extremities or aortic aneurism
- atrial fibrillation or recent MI

Invasive tests and treatments

- Tests
  - biopsy
  - retrograde pyelography
  - transfemoral arteriography

- Treatments
  - steroids
  - conservative therapy
  - iv-fluids
  - surgery for urinary tract obstruction
  - antibiotics
  - surgery for clot in renal vessels
  - antihypertensive drugs
  - heparin

Updating probability distribution

\[ P_{i+1}(D_j) = \frac{P_i(D_j)P(S|D_j)}{\sum_{k=1}^{n} P_i(D_k)P(S|D_k)} \]

Bayes’ rule

Value of treatment

- Three results: improved, unchanged, worsened
  - each has an innate value, modified by “tolls” paid on the way

- Probabilities depend on underlying disease probability distribution
Modeling treatment

Utilities:
- improved: 5000
- unchanged: -2500
- worse: -5000

Modeling test: transfemoral arteriography

<table>
<thead>
<tr>
<th>Steroids</th>
<th>improved</th>
<th>unchanged</th>
<th>worse</th>
</tr>
</thead>
<tbody>
<tr>
<td>atn</td>
<td>0.60</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>farf</td>
<td>0.05</td>
<td>0.35</td>
<td>0.60</td>
</tr>
<tr>
<td>obstr</td>
<td>0.05</td>
<td>0.60</td>
<td>0.35</td>
</tr>
<tr>
<td>agn</td>
<td>0.40</td>
<td>0.40</td>
<td>0.20</td>
</tr>
<tr>
<td>cn</td>
<td>0.05</td>
<td>0.75</td>
<td>0.20</td>
</tr>
<tr>
<td>hs</td>
<td>0.05</td>
<td>0.06</td>
<td>0.90</td>
</tr>
<tr>
<td>pye</td>
<td>0.05</td>
<td>0.05</td>
<td>0.90</td>
</tr>
<tr>
<td>ae</td>
<td>0.05</td>
<td>0.70</td>
<td>0.25</td>
</tr>
<tr>
<td>ri</td>
<td>0.01</td>
<td>0.14</td>
<td>0.85</td>
</tr>
<tr>
<td>rvt</td>
<td>0.10</td>
<td>0.30</td>
<td>0.60</td>
</tr>
<tr>
<td>vasc</td>
<td>0.15</td>
<td>0.25</td>
<td>0.60</td>
</tr>
<tr>
<td>scl</td>
<td>0.05</td>
<td>0.05</td>
<td>0.90</td>
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<td>cgaee</td>
<td>0.40</td>
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<td>0.25</td>
</tr>
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<td>mh</td>
<td>0.05</td>
<td>0.05</td>
<td>0.90</td>
</tr>
</tbody>
</table>

How large is the tree?

- Infinite, or at least \((27+3+8)^{(27+3+8)}\), \(\sim 10^{60}\)
- What can we do?
  - Assume any action is done only once
    - Order:
      - questions
      - tests
      - treatments
- \(27! \times 4 \times 3 \times 2 \times 8\), \(\sim 10^{30}\)
- Search, with a myopic evaluation function
  - like game-tree search; what’s the static evaluator?
  - Measure of certainty in the probability distribution

How many questions needed?

- How many items can you distinguish by asking 20 (binary) questions? \(2^{20}\)
- How many questions do you need to ask to distinguish among \(n\) items? \(\log_2(n)\)
- Entropy of a probability distribution is a measure of how certainly the distribution identifies a single answer; or how many more questions are needed to identify it
Entropy of a distribution

\[ H(p_1, \ldots, p_n) = \sum_{j=1}^{n} -p_j \log_2 p_j \]

For example:
- \( H(.5, .5) = 1.0 \)
- \( H(.1, .9) = 0.47 \)
- \( H(.01, .99) = 0.08 \)
- \( H(.001, .999) = 0.01 \)
- \( H(.33, .33, .33) = 1.58 (!) \)
- \( H(.005, .455, .5) = 1.04 \)
- \( H(.005, .995, 0) = 0.045 \)

(!) -- should use \( \log_n \)

Interacting with ARF in 1973

Question 1: What is the patient's age?
1. 0-10
2. 11-30
3. 31-50
4. 51-70
5. Over 70
Reply: 5

The current distribution is:

<table>
<thead>
<tr>
<th>Disease</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>FARF</td>
<td>0.58</td>
</tr>
<tr>
<td>IBSTR</td>
<td>0.22</td>
</tr>
<tr>
<td>ATN</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Question 2: What is the patient's sex?
1. Male
2. Pregnant Female
3. Non-pregnant Female
Reply: 1

ARF in 1994

Local Sensitivity Analysis
Case-specific Likelihood Ratios

Therapy Planning Based on Utilities

Assumptions in ARF

- Exhaustive, mutually exclusive set of diseases
- Conditional independence of all questions, tests, and treatments
- Cumulative (additive) disutilities of tests and treatments
- Questions have no modeled disutility, but we choose to minimize the number asked anyway