Announcements

Problem Set 5 – Posted on Stellar. Due Wed, Oct. 10

Qualitative description - MOS in thermal equilibrium

Definition of structure: metal/silicon dioxide/p-type Si (Example: n-MOS)
Electrostatic potential of metal relative to silicon: $\phi_m$
Zero bias condition: Si surface depleted if $\phi_m > \phi_{p-Si}$ (typical situation)
Negative bias on metal: depletion to flat-band to accumulation
Positive bias on metal: depletion to threshold to inversion

Quantitative modeling - MOS in thermal equilibrium, $v_{BC} = 0$

Depletion approximation applied to the MOS capacitor:
1. Flat-band voltage, $V_{FB}$
2. Accumulation layer sheet charge density, $q_{A^*}$
3. Maximum depletion region width, $X_{DT}$
4. Threshold voltage, $V_T$
5. Inversion layer sheet charge density, $q_{N^*}$

Quantitative modeling - $v_{BC} \neq 0$; impact of $v_{BC} < 0$

Voltage between n+ region and p-substrate: $|2\phi_{p-Si}| \rightarrow |2\phi_{p-Si}| - v_{BC}$
**n-Channel MOSFET:** Connecting with the npn MOSFET

A very similar behavior, and very similar uses.

\[ i_D \approx K [v_{GS} - V_T(v_{BS})]^2 / 2\alpha \]

\[ i_B \approx I_{BSE} \frac{q}{kT} \]

\[ v_{CE} > 0.2 \text{ V} \]
An n-channel MOSFET

In an n-channel MOSFET, we have two n-regions (the source and the drain), as in the npn BJT, with a p-region producing a potential barrier for electrons between them. In this device, however, it is the voltage on the gate, $v_{GS}$, that modulates the potential barrier height.

The heart of this device is the MOS capacitor, which we will study today. To analyze the MOS capacitor we will use the same depletion approximation that we introduced in conjunction with p-n junctions.
The n-MOS capacitor

Right: Basic device with $v_{BC} = 0$

Below: One-dimensional structure for depletion approximation analysis*

* Note: We can't forget the n+ region is there; we will need electrons, and they will come from there.
Electrostatic potential and net charge profiles

Zero bias: $v_{GB} = 0$

\[ \phi(x) \]

\[ \rho(x) \]

$-t_{ox}$

$x_d$

$\phi_p$

$\phi_m$

$q_{D}^* = -qN_A x_d$

$qN_A x_d$

$-qN_A$

$-t_{ox}$
Electrostatic potential and net charge profiles

Depletion: $V_{FB} < v_{GB} < 0$

\[ \phi(x) \]

\[ \phi(x) = \phi_p - qN_A x_d \]

\[ \rho(x) \]

\[ \rho(x) = -qN_A \]

$v_{GB} < 0$
Electrostatic potential and net charge profiles

\[ \phi(x) \]

Flat band: \( V_{GB} = V_{FB} \)

\[ V_{FB} = \phi_p - \phi_m \]

\[ V_{FB} = \phi_p - \phi_m \]
Electrostatic potential and net charge profiles

Accumulation: $v_{GB} < V_{FB}$

\[ \phi(x) \]

\[ \rho(x) \]

$-t_{ox}$

$v_{GB} < V_{FB}$

$-t_{ox}$

$-C_{ox}^*(v_{GB} - V_{FB})$

$C_{ox}^*(v_{GB} - V_{FB})$
Electrostatic potential and net charge profiles

Flat band: $v_{GB} = V_{FB}$
Electrostatic potential and net charge profiles

Depletion: $V_{FB} < V_{GB} < 0$

\[ \phi(x) \]

\[ \rho(x) \]

$V_{FB} < V_{GB} < 0$

$qN_A x_d$

$-t_{ox}$

$x_d$

$\phi_p$

$\phi_m$

$\rho(x)$
Electrostatic potential and net charge profiles

Depletion: $v_{GB} = 0$

\[ \phi(x) \]

\[ \phi_m \]

\[ \phi_p \]

\[ -t_{ox} \]

\[ x_d \]

\[ \rho(x) \]

\[ q_N A x_d \]

\[ -t_{ox} \]

\[ -q_N A \]

\[ q_D^* = -q_N A x_d \]
Electrostatic potential and net charge profiles

Depletion: $0 < V_{GB} < V_T$
Weak inversion: $\phi(0) > 0$

\[ J = 0 \implies n(x) = n_i e^{-q\phi(x)/kT} \]
and $p(x) = n_i e^{q\phi(x)/kT}$

$\phi(0) \uparrow \implies n(0) \uparrow$

Weak inversion: $\phi(0) > 0 \implies n(0) > p(0)$
Electrostatic potential and net charge profiles

Threshold: \( V_{GB} = V_T \)

At threshold \( \phi(0) = -\phi_p \)

\[ \phi(0) = -\phi_p \Rightarrow n(0) = N_A \]

\[ qD^* = -qN_A X_{DT} \]
Threshold*: $v_{GB} = V_T$

$$X_{DT} = (2\varepsilon_S l|2\phi_p|/qN_A)^{1/2}$$

$$V_T - V_{FB} = |2\phi_p| + qN_A X_{DT}/C_{ox}^*$$

$$V_T = V_{FB} + |2\phi_p| + (2\varepsilon_S |2\phi_p|/qN_A)^{1/2}/C_{ox}^*$$

* At threshold $\phi(0) = -\phi_p$
Inversion: $V_T < V_{GB}$

- $\phi(x)$
- $\phi_p$
- $\phi_m$
- $X_{DT}$
- $2\phi_p$
- $q_N^* = \text{Inversion layer charge (sheet of mobile electrons in Si near the Si-oxide interface)}$
- $q_D^*$, depletion region charge unchanged

$q_N^* = -qN_A X_{DT}$

$q_N^* = -C_{ox}^*(V_{GB} - V_T)$

$q_{D}^* = -qN_A X_{DT}$

$q_{N}^* = -C_{ox}^*(V_{GB} - V_T)$

Electrostatic potential and net charge profiles

$V_T < V_{GB}$

$-t_{ox}$
Electrostatic potential and net charge profiles - *regions and boundaries*

**Accumulation**

\( V_{GB} < V_{FB} \)

**Depletion**

\( V_{FB} < V_{GB} < V_T \)

**Inversion**

\( V_T < V_{GB} \)

**Flat Band Voltage**

\( V_{FB} = \phi_p - \phi_m \)

**Threshold Voltage**

\[ V_T = V_{FB} + |2\phi_p| + (2\varepsilon_{Si} |2\phi_p| q_N)^{1/2}/C_{ox}^* \]
Electrostatic potential and net charge profiles
- the grand procession from accumulation to inversion -

Accumulation: \( v_{GB} < V_{FB} \)

\[ \phi(x) \]

\[ \rho(x) \]

\[ -C_{ox}^*(v_{GB} - V_{FB}) \]

\[ -t_{ox} \]

\[ -\phi_p \]

\[ 0 \]

\[ V_{FB} \]

\[ V_{GB} < V_{FB} \]

\[ v_{GB} < V_{FB} \]
Electrostatic potential and net charge profiles
- the grand procession from accumulation to inversion -

Flat band: \( V_{GB} = V_{FB} \)
\( \phi(0) = \phi_p \)

\[ V_{FB} = \phi_p - \phi_m \]
Electrostatic potential and net charge profiles
- the grand procession from accumulation to inversion -

Depletion: $V_{FB} < V_{GB} < 0$
$\phi_p < \phi(0)$
Electrostatic potential and net charge profiles
- the grand procession from accumulation to inversion -

Depletion: $v_{GB} = 0$
$\phi_p < \phi(0) < 0$

\[ q_N A x_d \]

\[ q_D^* = -q_N A x_d \]
Electrostatic potential and net charge profiles
- the grand procession from accumulation to inversion -

Depletion: $0 < v_{GB} < V_T$
Weak Inversion: $\phi(0) > 0$
Electrostatic potential and net charge profiles
- the grand procession from accumulation to inversion -

Threshold: \( V_{GB} = V_T \)
\( \phi(0) = -\phi_p \)

\[ V_T = V_{FB} + |2\phi_p| + (2\varepsilon_{Si}|2\phi_p|qN_A)^{1/2}/C_{ox}^* \]
**Electrostatic potential and net charge profiles**
- the grand procession from accumulation to inversion -

**Inversion:** $V_T < v_{GB}$

- $\phi_p < \phi(0)$

$V_{TB}$

$V_T < v_{GB}$

$V_{FB}$

$0$

$rho(x)$

$qN_A X_{DT}$ + $C_{ox}^*(v_{GB} - V_T)$

$-t_{ox}$

$-qN_A$

$q_D^* = -qN_A X_{DT}$

$q_N^* = -C_{ox}^*(v_{GB} - V_T)$
Bias between n+ region and substrate

Reverse bias applied to substrate, i.e. \( v_{BC} < 0 \)

Now electrons under the gate will be attracted to the n+ region; we no longer have zero current and an electron population in TE with \( \phi(x) \).

Soon we will see how this will let us electronically adjust MOSFET threshold voltages when it is convenient for us to do so.
With voltage between substrate and channel, $v_{BC} < 0$

Flat band: $v_{GB} = V_{FB}$

No difference from when $v_{BC} = 0$
With voltage between substrate and channel, $v_{BC} < 0$

Depletion: $0 < v_{GB} < V_T(v_{BC})$

No difference from when $v_{BC} = 0$

except electrons don’t flow from the n+ region in under the electrode.*

Strictly speaking, we don’t have TE.

* Not a big deal since they contribute so little to the total charge, but it will matter nearer $V_{TH}$.
With voltage between substrate and channel, \( v_{BC} < 0 \)

Depletion: \( 0 < v_{GB} < V_T(v_{BC}) \)

Still looks like the \( v_{BC} = 0 \) case, but electrons still can’t flow from the n+ region into the channel!
With voltage between substrate and channel, $v_{BC} < 0$

At threshold: $v_{GB} = V_T(v_{BC})$

Big difference from when $v_{BC} = 0$.

Only now can electrons flow from the n+ region and under the gate. $V_T$ is larger than when $v_{BC} = 0$. 

$X_{DT}(v_{BC} < 0)$
With voltage between substrate and channel, $v_{BC} < 0$

Threshold: $v_{GB} = V_T(v_{BC})$ with $v_{BC} < 0$

$V_T(v_{BC}) = V_{FB} + |2\phi_p| - v_{BC} + [2\varepsilon_Si(|2\phi_p| - v_{BC})qN_A]^{1/2}/C_{ox}^*$

{Note: This is $v_{GB}$ at threshold; to get $v_{GC}$ at threshold we add $v_{BC}$.}

$X_{DT}(v_{BC} < 0) = [2\varepsilon_Si(|2\phi_p| - v_{BC})/qN_A]^{1/2}$

$q_N^* = -qN_A X_{DT}$

$q_N^* = -[2\varepsilon_Si(|2\phi_p| - v_{BC})qN_A]^{1/2}$
• Qualitative description

  Three surface conditions: accumulated, depleted, inverted
  Two key voltages: flat-band voltage, $V_{FB}$; threshold voltage, $V_T$
  The progression: accumulation through flat-band to depletion, then depletion through threshold to inversion

• Quantitative modeling

  Apply depletion approximation to the MOS capacitor, $v_{BC} = 0$
  Definitions:
  $V_{FB} \equiv v_{GB}$ such that $\phi(0) = \phi_{p-Si}$
  $V_T \equiv v_{GB}$ such that $\phi(0) = -\phi_{p-Si}$
  $C_{ox}^* \equiv \epsilon_{ox}/t_{ox}$

  Results and expressions  (For n-MOS example)
  1. Flat-band voltage, $V_{FB} = \phi_{p-Si} - \phi_m$
  2. Accumulation layer sheet charge density, $q_A^* = -C_{ox}^*(v_{GB} - V_{FB})$
  3. Maximum depletion region width, $X_{DT} = [2\epsilon_{Si}(|2\phi_{p-Si} - v_{BC}|/qN_A)^{1/2}$
  4. Threshold voltage, $V_T = V_{FB} - 2\phi_{p-Si} + [2\epsilon_{Si} qN_A(|2\phi_{p-Si} - v_{BC})]^{1/2}/C_{ox}^*$
  5. Inversion layer sheet charge density, $q_N^* = -C_{ox}^*(v_{GB} - V_T)$
npn BJT: The Ebers-Moll model

The forward model is what we use most, but adding the reverse model we cover the entire range of possible operating conditions.

Forward:

\[ I_{ES} = A q n^2 \left( \frac{D_h}{N_{DE} w_{E,eff}} + \frac{D_e}{N_{AB} w_{B,eff}} \right) \]

\[ \beta_F = \frac{(1 - \delta_B)}{(\delta_E + \delta_B)} \]

\[ \alpha_F = \frac{(1 - \delta_B)}{(1 + \delta_E)} \]

\[ \delta_E = \frac{i_{BE}}{i_{eE}} = \frac{D_h N_{AB}}{D_e N_{DE} w_{E,eff}} \]

\[ \delta_B \approx \frac{w_{B,eff}^2}{2D_e \tau_e} = \frac{w_{B,eff}^2}{2L_e^2} \]

Combined they form the full Ebers-Moll model:

\[ i_E(v_{BE},v_{BC}) = i_{EF}(v_{BE},0) + i_{ER}(0,v_{BC}) \]

\[ i_C(v_{BE},v_{BC}) = i_{CF}(v_{BE},0) + i_{CR}(0,v_{BC}) \]

\[ i_B(v_{BE},v_{BC}) = i_{BF}(v_{BE},0) + i_{BR}(0,v_{BC}) \]

Reverse:

\[ I_{CS} = A q n^2 \left( \frac{D_h}{N_{DC} w_{C,eff}} + \frac{D_e}{N_{AB} w_{B,eff}} \right) \]

\[ \beta_R = \frac{(1 - \delta_B)}{(\delta_C + \delta_B)} \]

\[ \alpha_R = \frac{(1 - \delta_B)}{(1 + \delta_C)} \]

\[ \delta_C = \frac{i_{BC}}{i_{eC}} = \frac{D_h N_{AB}}{D_e N_{DC} w_{C,eff}} \]

\[ \delta_B \approx \frac{w_{B,eff}^2}{2D_e \tau_e} = \frac{w_{B,eff}^2}{2L_e^2} \]

Note: \( i_F = -i_E(v_{BE},0) \)

and \( i_R = -i_C(0,v_{BC}) \).

You are not responsible for this model.
nnp BJT: The Gummel-Poon model

Another common model can be obtained from the Ebers-Moll model is the Gummel-Poon model:

\[ I_S = \frac{\beta_F}{(\beta_F + 1)} I_{ES} = \frac{\beta_R}{(\beta_R + 1)} I_{CS} \]

\[ = \alpha_F I_{ES} = \alpha_R I_{CS} \]

Combined they form the Gummel-Poon model:

Aside from the historical interest, another value this has for us in 6.012 is that it is an interesting exercise to show that the two forward circuits above are equivalent.

You even less responsible for this model.