Yesterday we introduced differential amplifiers. Very often the signal we want to measure & amplify are buried inside noises. Using differential amplifiers we can pick out the signal but reject the common background noises.

Today we will look at more detail of the LEC of DA. Our lecture notes covered MOSFET, we will use BJT here as an example, to be complimentary.

A DA using BJT:

Q₁ & Q₂ are matched

Assume we have two signals \( X₁, X₂ \) (either voltage or current)

Differential mode signal: \( X_{DM} = X₁ - X₂ \)

Common mode signal: \( X_{CM} = \frac{X₁ + X₂}{2} \)

with this any inputs can be decomposed into

\[
\begin{align*}
V_{i1} &= V_{ic} + \frac{V_{id}}{2} \\
V_{i2} &= V_{ic} - \frac{V_{id}}{2}
\end{align*}
\]

where

\[
V_{ic} = \frac{V_{i1} + V_{i2}}{2} \quad V_{id} = V_{i1} - V_{i2}
\]

with the small signal circuit of our DA being linear (LEC), if we know the common mode gain \( (A_{CM} = \frac{V_{oC}}{V_{ic}} = \frac{V_{o1} + V_{o2}}{2}) \) and differential mode gain \( (A_{DM} = \frac{V_{oD}}{V_{id}} = \frac{V_{o1} - V_{o2}}{2}) \), we can find the output signals by linear superposition.

\[
\begin{align*}
V_{o1} &= V_{oC} + \frac{V_{oD}}{2} = A_{CM} \cdot V_{ic} + A_{DM} \frac{V_{id}}{2} \\
V_{o2} &= V_{oC} - \frac{V_{oD}}{2} = A_{CM} \cdot V_{ic} - A_{DM} \frac{V_{id}}{2}
\end{align*}
\]

1. Small signal model of purely differential mode input:

\[
\begin{align*}
\frac{V_{oD}}{V_{id}} = \frac{1}{G_{ds}} \text{ (here) } G_{ds} \text{ can be easily added if needed}
\end{align*}
\]

\( G_{ds} \) \rightarrow \text{ internal resistance of } I_{BIAS}
since $Q_1$ & $Q_2$ are matched, $V_{R1} = V_{R2} = V_{T}$, $G_m = G_m = G_m$, $g_{o1} = g_{o2} = g_o$

due to the symmetry of the circuit we argue that $i_1 = -i_2$

as a result, the current flow through $g_{sc}$ should be zero, $V_X$ is at virtual ground.

Then we can look at only half of the circuit equivalently:

\[ V_{in} = \frac{V_{id}}{2}, \quad V_{o1} = -\frac{g_m V_{id}}{g_o + g_{sc}} \Rightarrow V_{o2} = \frac{-g_m \frac{V_{id}}{2}}{g_o + g_{sc}} \]

\[ V_{od} = V_{o1} - V_{o2} = -\frac{g_m V_{id}}{g_o + g_{sc}} \Rightarrow AV_d = \frac{V_{od}}{V_{id}} = -\frac{g_m}{g_o + g_{sc}} \quad \text{(same as the one for MOSFET)} \]

2. Small signal model of purely common-mode input:

due to the symmetry of the circuit, we argue that $i_1 = i_2 = i$

Therefore $V_X = (i_1 + i_2)/g_{sc} = 2i/g_{sc} = i/g_{sc}$

 equivalently we can split the circuit in half again

This configuration is the common-emitter amplifier with an emitter degeneration resistor (similar to the C-S amplifier with a source-degeneration resistor).
the current through $g_{ss}/2$ come from $\beta C_e$ (which is $\beta C$ of BJT), $g_{mC}$, $g_C$

\[
\begin{align*}
\eta & = \frac{\beta C}{\beta C_e} = \frac{g_m}{g_{mC}} = \frac{1}{\beta C_e} \\
\eta & = \frac{g_m}{g_{mC}} \ll \frac{g_mV}{C_e} \\
\eta & \text{ is relatively small (so we ignore for the 1st order)}
\end{align*}
\]

\[
\frac{V_{in}}{V_{in}} \rightarrow \text{ (similar to MOSFET, MOSFET } \frac{R_{in}}{V_{in}} = \infty)
\]

\[
\begin{align*}
V_{ic} & = V_{ic} + \frac{g_mV_{ic}}{g_{ss}/2} , \quad V_{o1} = -\frac{g_mV_{ic}}{g_{sl}} \\
\frac{V_{o1}}{V_{ic}} = \frac{V_{o2}}{V_{ic}} & = \frac{-g_m}{1 + g_m/g_{ss}} \quad \text{since } g_m \gg g_{ss}, \quad \frac{V_{o1}}{V_{ic}} = \frac{V_{o2}}{V_{ic}} \approx \frac{g_m}{2}\frac{g_{ss}}{g_{ss}}
\end{align*}
\]

\[
\text{Common mode gain} = \frac{V_{oc}}{V_{ic}} = \frac{(V_{o1} + V_{o2})/2}{V_{ic}} \approx \frac{g_m}{2}\frac{g_{ss}}{g_{ss}}
\]

\[
\text{Common mode rejection ratio: } \text{CMRR} = \frac{\text{Auc}}{\text{Auc}} = \frac{V_{oc}/V_{id}}{V_{oc}/V_{id}} = \frac{2g_m}{g_{ss}}
\]

**Example: Analysis of a DA:**

A DA driven by a single ended voltage source, the output voltage is taken from the right hand side as a single ended voltage (i.e., reference to ground).

\[
\begin{align*}
V^+ & \quad 10k \Omega \quad 3 \quad 10k \Omega \\
K & \quad V_{in} \quad 500 \mu A \quad 500 \mu A \\
& \quad 5k \Omega \quad 500 \mu A \\
& \quad 500 \mu A \\
& \quad \frac{V_{in}}{2} = V_{in} = V_{v1}, V_{v2} = 0 \\
\Rightarrow V_{id} = V_{v1} - V_{v2} = V_{v1} = (V_{v1} + V_{v2})/2 = V_{v1}/2
\end{align*}
\]

\[
\begin{align*}
g_m & = \frac{g_{mC}}{2} \cdot \frac{I_{BAS} \cdot 25 kV}{500 \mu A} = 10 mS \\
A_{vd} & = -\frac{g_m}{g_{v1} + g_{v2}} = -\frac{g_m}{g_{v1} + g_{v2}} = -90 mV (\text{ohm}) = -100 \\
A_{uc} & = -\frac{g_{ss}}{2g_{sl}} = -\frac{2g_{ss}}{2g_{sl}} = -100 \\
V_{oc} & = V_{oc} - V_{oc}/2
\end{align*}
\]

\[
\begin{align*}
V_{oc} & = \text{Auc} \cdot V_{ic}, V_{oc} = \text{Auc} \cdot V_{id}, V_{oc} = \text{Auc} \cdot V_{ic} - \text{Auc} \cdot V_{id} = \text{Auc} \cdot V_{ic} - \text{Auc} \cdot V_{id} \\
V_{oc} & \Rightarrow V_{oc} = \frac{V_{oc}}{V_{in}} \approx \frac{50}{2}
\end{align*}
\]
The differential gain was reduced by a factor of 2 because we used a single ended output. CMRR and CMRR is 1000.