Yesterday we started the discussion on Frequency response of amplifier circuits (using C.S. Amplifier as an example.)

Today we will have a review of yesterday's discussion, we will use Common Emitter amplifier as an example.

We will use the following method to find the high frequency limit:

1. Nodal analysis (full treatment, derive \( \frac{V_{out}}{V_{in}} \) or \( V_s \).

2. Miller effect & Miller Approximation

3. Open Circuit Time constant technique.

\[ g_t = g_o + g_c \]

(when considering high freq limit, \( C_o, C_{ce} \) becomes short)

Freq response is to see how \( \frac{V_{out}}{V_{in}} \) depend on freq.

1. Nodal Analysis:

Node 1: \((V_t - V_C) g_t - V_C g_c - j\omega C_v V_C - j\omega C_m V_{in} - V_{out}(V_C - V_{out}) = 0 \)  

Node 2: \( j\omega C_v (V_C - V_{out}) - g_m V_C - V_{out}(g_o + g_c) = 0 \)

From (2) \[ V_C (j\omega C_v - G_m) = V_{out}(g_o + g_c + j\omega C_m) \] plug

\[ V_C g_c - (g_o + g_c + j\omega C_v + j\omega C_m) V_C + j\omega C_m V_{out} = 0 \]

\[ V_C = \frac{(g_o + g_c + j\omega C_v) V_{out} + j\omega C_m V_{out}}{j\omega C_v - G_m} \]

\[ V_{out} = \frac{-g_o(G_m - j\omega C_m)}{j\omega C_v (G_v + (j\omega) (G_o + g_c) G_v + (g_o + g_c + j\omega C_v + j\omega C_m) G_v) + (G_o + g_c) (G_o + g_c)} \]

This is very similar result as what we get yesterday for C-S, with \( g_o \rightarrow 0 \), \( C_o \rightarrow \infty \), \( C_e \rightarrow \infty \)
This gain can be expressed as:

\[
\frac{V_{out}}{V_i} = \frac{A_{v,LF} (1 + j\omega T_3)}{(1 + j\omega T_1) (1 - j\omega T_2)} = \frac{A_{v,LF} (1 - j\omega T_2)}{1 - j\omega (T_1 + T_2) + (j\omega)^2 T_1 T_2}
\]

\[A_{v,LF} \text{ is low frequency gain:}
\]

\[
= \frac{-g_m}{g_0 + g_i} \frac{g_e}{g_t + g_m}
\]

\[\text{two poles } (\omega_1 = 1/T_1, \omega_2 = 1/T_2) \quad \text{and one-zero } (\omega_3 = 1/T_3)
\]

\[T_1 \gg T_2, T_3 \quad \text{i.e. } \omega_1 < \omega_2, \omega_3
\]

what we really care is where \(\omega_1\)

at what freq does the gain begin to fall off? \(\text{dB} = 20 \times \log\)

\[3\text{dB: } \omega_1 = 1
\]

It is hard to solve \(T_1, T_2\) or \(\omega_1, \omega_2\), but \(T_1 \gg T_2, T_1 + T_2 \approx T_1\)

\[\Rightarrow \omega_{dB} \approx \sqrt{T_1 + T_2} = \left[\frac{1}{(g_0 + g_i)(g_t + g_m)} \cdot \left(\frac{g_0 + g_i}{g_t + g_m} + (g_0 + g_t + g_e + g_i + g_m) \cdot g_i\right)\right]^{-1}
\]

or \(\omega_{HI}\) as presented in lecture notes

2. Miller Approximation,

\[C_M = g_i (1 - A_{v,LF}) \quad \Rightarrow \quad \text{\(g_i\) being amplified by the gain}
\]

\[V_M = V_i \cdot \frac{1}{g_0 + j\omega (g_0 + g_m)} + \frac{1}{g_t + j\omega (g_t + g_m)} = \left(\frac{g_e}{g_t + g_i + j\omega (g_t + g_m)}\right) \cdot V_i
\]

\[V_{out} = -\frac{g_m V_M}{g_0 + g_i} \quad \Rightarrow \quad V_{out} = -\frac{g_m g_i}{(g_0 + g_i) (g_t + g_m) + (g_0 + g_i + g_m) g_i}
\]

\[3\text{dB} \approx \left[\frac{1}{(g_0 + g_i) (g_t + g_m) + (g_0 + g_t + g_m) g_i}\right]^{-1}
\]
3. Open Circuit Time Constant (OCTC)

Assumptions:
1) No zeros (or zeros can be ignored)
2) One dominant pole (\(T_1 \gg T_2, T_3 \ldots\))

Procedures:
1) Open circuit all capacitors
2) Turn off all independent sources, find Thevenin resistance for each capacitor
3) Sum up: \(R_{th} C_i = \omega_{3dB} = (\Sigma R_{th} C_i)^{-1}\)

\(1)\ C_{th}\:

\[R_{th} = K_{th} R_C = (g_t + g_m)^{-1}\]

\(2)\ C_{th}\:

\[
\dot{V}_C = -V_C (g_t + g_m), \quad \dot{V}_C = g_m V_C + (V_C + V_R) (g_o + g_c)
\]

Plug (1) \(V_C\) into (2),

\[
\dot{V}_C = g_m \frac{-\dot{V}_C}{g_t + g_m} + (V_C + \frac{\dot{V}_C}{g_t + g_m}) (g_o + g_c)
\]

\[
(g_t + g_m + g_m + g_o + g_c), \quad \dot{V}_C = V_C (g_o + g_c) (g_t + g_m)
\]

\[
R_{th} = \frac{V_C}{\dot{V}_C} = \frac{(g_t + g_m + g_m + g_o + g_c)}{(g_o + g_c) (g_t + g_m)}
\]

\[
\omega_{3dB} = (\Sigma R_{th} C_i)^{-1} = \left(\frac{1}{\frac{1}{g_t + g_m} + \frac{g_t + g_m + g_o + g_c}{(g_o + g_c) (g_t + g_m)}} + C_u^{-1}\right)^{-1}
\]

\[
= \left[\frac{1}{(g_t + g_m) (g_o + g_c)} \left( (g_o + g_c) C_o + (g_o + g_c + g_t + g_m + g_o + g_c) C_u \right) \right]^{-1}
\]