Problem 1:

a) In saturation, \( i_D = (W/L) \mu_c (\varepsilon_{ox}/t_{ox})(V_{GS} - V_T)^2/2 = 3.33 \times 10^3 \times 3.5 \times 10^{-7} \times 2 = 2.33 \text{ mA} \)

b) Being in saturation requires that \( V_{DS} \geq (V_{GS} - V_T) = (3 - 1) = 2 \text{ Volts} \)

c) \( q_N^* (y) = C_{ox}^* [V_{GC}(y) - V_T] \) with \( C_{ox}^* = \varepsilon_{ox}/t_{ox} = 3.5 \times 10^{-7} \text{ Coul/V-cm}^2 \) and \( V_T = 1 \text{ V} \)

\( V_{GC}(0) = V_{GS} = 3 \text{ V} \); thus \( q_N^*(0) = C_{ox}^* [V_{GC} - V_T] = 3.5 \times 10^{-7} \times 2 = 7.0 \times 10^{-7} \text{ Coul/cm}^2 \)

\( V_{GC}(L) = V_{GD} = V_{GS} - V_{DS} = 2 \text{ V} \); thus \( q_N^*(L) = C_{ox}^* [V_{GD} - V_T] = 3.5 \times 10^{-7} \times 1 = 3.5 \times 10^{-7} \text{ C/cm}^2 \)

d) \( \tilde{s}_n(y) = I_D/[W \times q_n^*(y)] \) with \( I_D = 8.8 \times 10^{-4} \text{ Coul/s and } W = 10 \mu \text{m} = 10^{-3} \text{ cm} \)

At \( y = 0 \), \( q_N^*(0) = 7.0 \times 10^{-7} \text{ Coul/cm}^2 \); thus \( \tilde{s}_n(0) = 8.8 \times 10^{-4}/7.0 \times 10^{-10} = 1.26 \times 10^6 \text{ cm/s} \)

At \( y = L \), \( q_N^*(L) = 3.5 \times 10^{-7} \text{ Coul/cm}^2 \); thus \( \tilde{s}_n(L) = 8.8 \times 10^{-4}/3.5 \times 10^{-10} = 2.52 \times 10^6 \text{ cm/s} \)

e) This bias point is in saturation. There is no change at the source end and \( q_N^*(0) \) is unchanged from Part c), i.e. \( q_N^*(0) = C_{ox}^* [V_{GS} - V_T] = 3.5 \times 10^{-7} \times 2 = 7.0 \times 10^{-7} \text{ Coul/cm}^2 \)

The channel is pinched off at the drain end and in the simple model \( q_N^*(L) = 0. \)

f) Rearrange the equation used in Part d) to get \( q_N^*(y) = I_D/[W \times \tilde{s}_n(y)] \) with \( \tilde{s}_n(y) = s_{sat} = 10^7 \text{ cm/s} \). Thus \( q_N^*(y) = 2.33 \times 10^{-3}/[10^3 \times 10^7] = 2.33 \times 10^{-7} \text{ Coul/cm}^2 \). The electron sheet density is this divided by \( 1.6 \times 10^{19} \), or \( 1.43 \times 10^{12} \text{ electrons/cm}^2 \). To put this in perspective we could compare it to the sheet charge density in the depletion region, \( q_N^* x_{sat} \), but we don't have enough information to do this.

Problem 2:

a) \( t_{ox} = \varepsilon_{ox}/C_{ox}^* = 3.5 \times 10^{13}/2.3 \times 10^7 = 1.52 \times 10^6 \text{ cm} = 15.2 \text{ nm} \)

b) The substrate is n-type (p-channel) because the capacitance decreases as \( V_{GS} \) is made more negative than \( V_{FB} \), implying the surface is being depleted, not accumulated as it would be on a p-type substrate.

c) We were told initially that the doping level is \( 10^{17} \text{ cm}^3 \), so knowing now that the substrate is n-type it must be that \( N_D = 10^{17} \text{ cm}^3 \), and \( \phi_n = 7 \times 60 \text{ mV} = 0.42 \text{ V} \).

d) It is always true that \( V_{FB} = \phi_S - \phi_{Metal} \); so we have \( \phi_{Metal} = \phi_S - V_{FB} = 0.42 - (-0.13) = 0.55 \text{ V} \). Note: It looks like the gate "metal" is heavily doped polycrystalline Si.

e) At \( V_{GS} = 0 \) we are more positive than flat-band on an n-type substrate so the surface must be accumulated, i.e. more n-type with a sheet of electrons at the interface.

f) We started with zero charge on the gate at flat-band and have moved 0.13 Volts into accumulation at \( V_{GS} = 0 \). Thus there is a positive charge density on the gate:

\( q_G^* = C_{ox}^* (V_{GS} - V_{FB}) = 2.3 \times 10^7 \times 0.13 = 2.99 \times 10^8 \text{ Coul/cm}^2 \)
g) The minimum capacitance occurs at threshold, and we are told in the problem description that \( V_T = -1.7 \) V.

h) At threshold the depletion region supports a potential drop of \( 2 \phi_n = 0.84 \) V and thus \( x_D = [(2 \varepsilon_{Si} 2 \phi_n) / qN_D]^{1/2} = [2 \times 10^{-12} \times 0.84 / 1.6 \times 10^{-19} \times 10^{17}]^{1/2} = 10^{-5} \) cm = 0.1 \( \mu \)m.

i) The minimum capacitance corresponds to the oxide capacitance, \( 0.23 \) \( \mu \)F/cm\(^2\), in series with the depletion region capacitance, \( \varepsilon_{Si} / x_D = 10^{-12} / 10^{-5} = 0.1 \) \( \mu \)F/cm\(^2\). We have:

\[
C_{\text{min}} = (1/C_{\text{oX}} + 1/C_{\text{depl}})^{-1} = (1/0.23 + 1/0.1)^{-1} = 14.35^{-1} = 0.07 \mu \text{F/cm}^2.
\]

j) The net sheet charge density in the inversion layer is \( C_{\text{ox}} (V_{GS} - V_T) \), and we want to evaluate this when \( (V_{GS} - V_T) = 1 \) V. Thus \( q^* = 2.3 \times 10^{-7} \) Coul/cm\(^2\).

k) The charge on the gate equals this charge plus the charge in the inversion layer, \( q_{N D} x_D = 1.6 \times 10^{-19} \times 10^{17} \times 10^{-5} = 1.6 \times 10^{-7} \) Coul/cm\(^2\). The \( q_0^* \) is \( 2.3 \times 10^{-7} + 1.6 \times 10^{-7} = 3.9 \times 10^{-7} \) Coul/cm\(^2\).

Problem 3:

Part I:

a) We want the two transistors to have drain currents of the same magnitude when they are in saturation, and the problem statement and the symmetry of the circuit tell us that every other term in the current expressions are equal, so we must clearly have \( W_n = W_p = W \). Thus \( W_n/W_p = 1 \).

b) When \( V_{IN} = 0 \) and \( V_{OUT} = V_{DD} = 1 \) V, we have \( V_{DSn} = 1 \) V and \( V_{DSP} = 0 \) V. Thus:

\[
V_{Tn} = V_{To} - \delta V_{DSn} = 0.3 - 0.05 \times 1 = 0.25 \text{ V}
\]

\[
V_{Tp} = V_{To} - \delta V_{DSP} = -0.3 \text{ V}
\]

c) When \( V_{IN} = V_{DD} = 1 \) V and \( V_{OUT} = 0 \), we have \( V_{Dsn} = 0 \) V and \( V_{DSP} = -1 \) V. Thus:

\[
V_{Tn} = V_{To} - \delta V_{Dsn} = 0.3 \text{ V}
\]

\[
V_{Tp} = V_{To} - \delta V_{DSP} = -0.3 - 0.05 \times -1 = -0.25 \text{ V}
\]

d) The plot is found on the right (just lines):

The basic approach can be found in the Lecture 15 slides. The LEC is the same independent of whether the MOSFETs are long gate, short with velocity saturation and DIBL, or operating subthreshold:

The \( g_m \)’s and \( g_o \)’s depend on the terminal characteristics of the transistors and the bias point, which in this case is where both transistors are in...
saturation. We are given the drain current expressions in saturation and with them we can the g's:

\[
g_{mn} = \frac{dI_D}{dV_{GS}}|_{Q} = W C_{ox} s_{sat} = 10^4 \times 1.8 \times 10^6 \times 10^7 = 1.8 \times 10^{-3}\ S
\]

\[
g_{mp} = \frac{dI_P}{dV_{GS}}|_{Q} = W C_{ox} s_{sat} = 1.8 \times 10^{-3}\ S
\]

\[
g_{on} = \frac{dI_D}{dV_{DSn}}|_{Q} = \delta W C_{ox} s_{sat} = \delta g_{mn} = 0.05 \times 1.8 \times 10^{-3} = 9 \times 10^{-5}\ S
\]

\[
g_{op} = \frac{dI_P}{dV_{DSp}}|_{Q} = \delta W C_{ox} s_{sat} = \delta g_{mp} = 9 \times 10^{-5}\ S
\]

f) The slope is the voltage gain, \( A_v = v_{out}/v_{in} = -(g_{mn} + g_{mp})/(g_{on} + g_{op}) = -1/\delta = -20 \)

Part II:

a) With \( v_{IN} = 0 \), we have \( v_{GSn} = 0 \) and \( v_{DSn} \approx 0.18 \) V. Thus \( [1 - \exp(-qv_{DSn}/nkT)] \approx 1 \) and we can approximate \( i_{Dn} \) as \( I_s \exp[q(0.18 - 0.27)/nkT] \). We have \( V_Tn = 0.27 \) V, which is 3 x 90 mV, so the exponential term, \( \exp[q(-V_{Tn})/nkT] \), is \( 10^{-3} \) and thus \( i_{Dn} = I_s \times 10^{-3} = 10^{-3} \) nA = 1 pA = 10^{-6} \mu A.

b) We must have \( -i_{Dp} = i_{Dn} \) so it must be true that \( i_{Dp} = -1 \) pA.

c) With \( v_{IN} = 0.18 \) V, the drain current of the n-channel device increase by a factor of 100 times to 100 pA because the exponential term is now \( \exp[q(0.18 - 0.27)/nkT] \), which is \( 10^{-3} \) and thus \( i_{Dn} = I_s \times 10^{-3} = 10^{-3} \) nA = 1 pA = 10^{-6} \mu A.

d) The time is \( \Delta V \times C_{L}/i_{Dn} = 0.18 \) V x 0.1 pF/100 pA = \( 1.8 \times 10^{-4} \) s.

e) To calculate \( v_{SDp} \), we begin with \( i_{Dp} = -I_s \exp[q(v_{SGP} - |V_{Tp}|)/nkT] \times [1 - \exp(-qv_{SDp}/kT)] = -1 \) pA. Since \( v_{SGP} \approx 0.18 \) V, we have \( (v_{SGP} - |V_{Tp}|) = (0.18 - 0.27) = -90 \) mV. Thus the exponential factor \( \exp[q(v_{SGP} - |V_{Tp}|)/nkT] \) is \( 10^{-1} \). This is 100 times larger than it was in the case of \( i_{Dn} \), meaning \( |i_{Dp}| \) could be much larger than \( i_{Dn} \). What is different not is that now \( v_{SDp} \) is small and we can not say \( \exp(-qv_{SDp}/kT) \ll 1 \). If \( i_{Dp} \) is -1 pA, the factor \( [1 - \exp(-qv_{SDp}/nkT)] \) must be \( 10^{2} \), and thus \( v_{SDp} = -(nkT/q) \ln (0.99) \).
Exam Statistics

Average/Standard deviation:

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Class median: 77

Distribution to nearest 5:

Find yourself in this picture