Figure 1: General acoustic circuit element, illustrating the terminals of the element, the pressure $p$ across the element, the volume velocity $u$ through the element, and the node at the connection of two terminals from different elements.
Figure 2: The circuit representation and terminal characteristic of a volume velocity source with intrinsic volume velocity $u_s$. 
Figure 3: The circuit representation and terminal characteristic of a pressure source with intrinsic pressure $p_S$. 
Figure 4: The circuit representation and terminal characteristic of an acoustic resistance with resistance $R_A$. 

\[
p(t) = Pe^{st} \\
u(t) = Ue^{st} \\
P = R_A U \\
Z_R = R
\]
Figure 5: The circuit representation and terminal characteristic of an acoustic mass with mass $M_A$. 

\[
p(t) = M_A \frac{du(t)}{dt}
\]

\[
\begin{align*}
\mathbf{u}(t) &= U e^{st} \\
p(t) &= P e^{st} \\
P &= sM_A U \\
Z_M &= sM_A
\end{align*}
\]
\[ u(t) = C_A \frac{dp(t)}{dt} \]

\[ u(t) = U e^{st} \quad p(t) = P e^{st} \]

\[ P = \frac{1}{sC_A} U \]

\[ Z_C = \frac{1}{sC_A} \]

Figure 6: The circuit representation and terminal characteristic of an acoustic compliance with compliance \( C_A \).
Figure 7: An example of the application of Kirchoff’s Volume Velocity Law. For volume velocities with arbitrary time dependence, KUL applies to the values of $u_1$, $u_2$ and $u_3$ at each instant of time, $u_1 + u_2 - u_3 = 0$. If all volume velocities have the same $e^{st}$ time dependence, KUL is satisfied at each instant of time if it is satisfied by the values of the volume velocities at $t = 0$, $U_1$, $U_2$, and $U_3$, so that $U_1 + U_2 - U_3 = 0$. 

\[
\begin{align*}
  u_1 &= U_1 e^{st} \\
  u_2 &= U_2 e^{st} \\
  u_3 &= U_3 e^{st}
\end{align*}
\]
Figure 8: An example of the application of Kirchoff’s Pressure Law. For pressures with arbitrary time dependence, KPL applies to the values of $p_1$, $p_2$, and $p_3$ at each instant of time, $p_1 + p_3 - p_2 = 0$. If all pressures have the same $e^{st}$ time dependence, KPL is satisfied at each instant of time if it is satisfied by the values of the pressures at $t = 0$, $P_1$, $P_2$, and $P_3$ so that $P_1 + P_3 - P_2 = 0$. 

$p_1(t)=P_1e^{st}$  
$p_2(t)=P_2e^{st}$  
$p_3(t)=P_3e^{st}$
Figure 9: Example of an acoustic circuit consisting of a pressure source and two elements connected in series. All pressures and volume velocities are assumed to have the same $e^{st}$ time dependence.
\[ U_S + U_1 = 0 \]  \hspace{1cm} \text{Node A.}

\[ -U_1 + U_2 = 0 \]  \hspace{1cm} \text{Node B.}

\[ U_S + U_2 = 0 \]

\[ -U_S = U_1 = U_2 = U \]

\[ -P_S + P_1 + P_2 = 0 \]

\[ P_S = P_1 + P_2 \]

\[ P_1 = U_1 Z_1 = U Z_1 \]

\[ P_2 = U Z_2 \]
\[ P_S = U(Z_1 + Z_2) \]

\[ U = \frac{1}{Z_1 + Z_2} P_S \]

\[ P_1 = \frac{Z_1}{Z_1 + Z_2} P_S \]

\[ P_2 = \frac{Z_2}{Z_1 + Z_2} P_S \]

\[ \frac{P_1}{P_2} = \frac{Z_1}{Z_2} \]
\[ b_i = \sum_{j=1}^{M} \alpha_j P_{S_j} + \sum_{k=1}^{N} \beta_k U_{S_k} \]
Figure 10: Example of an acoustic circuit containing a port. It can be shown that Kirchoff’s volume velocity law requires that $u(t) = u'(t)$. 
Figure 11: An acoustic circuit for which it is possible to apply Thevenin’s Theorem.
\[ P = P_{Th} + Z_{Th}U_0 \]

Figure 12: The “Thevenin Equivalent” acoustic circuit at a pair of terminals.
Figure 13: An acoustic circuit for which it is possible to apply Norton’s Theorem.
\[ U = -U_N + \frac{P_0}{Z_N} \]

Figure 14: The Norton Equivalent acoustic circuit at a pair of terminals.
\[ P_{Th} = U_N Z_N \]
\[ Z_{Th} = Z_N \]
Figure 15: An acoustic circuit to which it is possible to connect two one-port circuits. In this case the one-port circuits are volume velocity sources with arbitrary intrinsic velocities $U_1$ and $U_2$. 
\[ P_1 = Z_{11}U_1 + Z_{12}U_2 + P_{OC_1} \]
\[ P_2 = Z_{21}U_1 + Z_{22}U_2 + P_{OC_2} \]

\[ U_1 = Y_{11}P_1 + Y_{12}P_2 - U_{SC_1} \]
\[ U_2 = Y_{21}P_1 + Y_{22}P_2 - U_{SC_2} \]

\[ P_1 = S_{11}U_1 + S_{12}P_2 + P_{OS_1} \]
\[ U_2 = S_{21}U_1 + S_{22}P_2 - U_{OS_2} \]
\[ P_1 = Z_{11} U_1 + Z_{12} U_2 \]
\[ P_2 = Z_{21} U_1 + Z_{22} U_2 \]

\[ U_1 = Y_{11} P_1 + Y_{12} P_2 \]
\[ U_2 = Y_{21} P_1 + Y_{22} P_2 \]

\[ P_1 = S_{11} U_1 + S_{12} P_2 \]
\[ U_2 = S_{21} U_1 + S_{22} P_2 \]
Figure 16: The acoustic network representation of a special type of two-port, the Ideal Transformer with a pressure ratio of $N$. 

\[ P_2 = N \quad P_1 \quad U_1 = -N \quad U_2 \]
\[
\frac{P_1}{U_2} \bigg|_{U_1=0} = \frac{P_2}{U_1} \bigg|_{U_2=0} \\
Z_{12} = Z_{21}
\]

\[
\frac{U_1}{P_2} \bigg|_{P_1=0} = \frac{U_2}{P_1} \bigg|_{P_2=0} \\
Y_{12} = Y_{21}
\]

\[
\frac{P_2}{P_1} \bigg|_{U_2=0} = -\frac{U_1}{U_2} \bigg|_{P_1=0} \\
S_{12} = -S_{21}
\]
\[ \frac{P}{U_0} = Z_{Th}(s) \]

\[ \frac{P_2}{U_1} \bigg|_{U_2=0} = Z_{21}(s) \]

\[ H(s) = \frac{n(s)}{d(s)} = K \frac{s^M + b_1 s^{M-1} + \cdots + b_M}{s^N + a_1 s^{N-1} + \cdots + a_N}, \]

\[ H(s) = K \frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)}. \]
\[ x(t) = X e^{st} \]
\[ y(t) = Y e^{st} \]

\[ 
H(j\omega) = \frac{Y}{X} 
\]
\[ x(t) = X \cos(\omega t) = \text{Re} [X e^{j\omega t}] = \text{Re} [X e^{j\omega t}] \]
\[ y(t) = Y \cos(\omega t + \theta) = \text{Re} [Y e^{j(\omega t + \theta)}] = \text{Re} [Y e^{j\omega t}] . \]

\[ X = X \]
\[ Y = Y e^{j\theta} = H(j\omega)X \]

\[ H(j\omega) = |H| e^{j\theta_H} \]

where
\[ |H| = \sqrt{\text{Re} [H]^2 + \text{Im} [H]^2} \]
\[ \theta_H = \arctan \frac{\text{Im} [H]}{\text{Re} [H]} . \]

\[ |Y| = |H| |X| = |H| X \]
\[ \theta = \theta_H , \]

\[ y(t) = \text{Re} [X |H| e^{j\theta_H} e^{j\omega t}] \]
\[ = X |H| \cos(\omega t + \theta_H) . \]
Figure 17: Circuit for Example 1. Assume that all pressures and volume velocities have an exponential ($e^{st}$) time dependence.
\[
H(s) = \frac{P_O}{P_S} = \frac{\omega_0}{\omega_0 + s}
\]

\[
Y(s) = \frac{U}{P_S} = \frac{1}{R} \frac{s}{\omega_0 + s}
\]

\[\omega_0 = 1/RC.\]

\[
H(j\omega) = \frac{\omega_0}{\omega_0 + j\omega} = \frac{1}{1 + j\Omega}
\]

\[\Omega = \omega/\omega_0 \text{ is a normalized frequency.}\]

\[
|H(j\omega)| = \frac{1}{\sqrt{1 + \Omega^2}}
\]

\[\theta_H = -\arctan \Omega\]
\[ Y(j\omega) = \frac{1}{R} \frac{j\omega}{\omega_0 + j\omega} = \frac{1}{R} \frac{j\Omega}{1 + j\Omega}. \]

\[ \frac{j\Omega}{R} = \frac{\Omega}{R} e^{j\pi/2}, \]

\[ Y(j\omega) = \frac{\Omega}{R} H(j\omega) e^{j\pi/2}. \]

\[ |Y(j\omega)| = \frac{1}{R} \frac{\Omega}{\sqrt{1 + \Omega^2}} \]

\[ \theta_Y = \frac{\pi}{2} - \arctan \Omega. \]
Figure 18: Dependence of the magnitude and phase of the input admittance system function $Y$ (solid curves) and pressure transfer ratio system function $H$ (dotted curves) on normalized frequency $\Omega = \omega/\omega_0$ for the acoustic circuit of Fig. 17. The magnitude of the admittance system function $Y$ has been multiplied by $R$. 
Figure 19: Dependence of the magnitude and phase of the input admittance system function $Y$ (solid curves) and pressure transfer ratio system function $H$ (dotted curves) on normalized frequency $\Omega = \omega/\omega_0$ for the acoustic circuit of Fig. 17. Log-Log coordinates are used. The magnitude of the admittance system function $Y$ has been multiplied by $R$. 
Figure 20: Circuit for Example 2. Assume that all pressures and volume velocities have an exponential ($e^{st}$) time dependence.
\[ H(s) = \frac{P_0}{P_s} = \frac{\omega_0^2}{s^2 + \alpha \omega_0 s + \omega_0^2}, \]

\[ Y(s) = \frac{U}{P_s} = \frac{1}{M s^2 + \alpha \omega_0 s + \omega_0^2}, \]

\[ \omega_0 = \frac{1}{\sqrt{MC}} \]

\[ \alpha = \frac{R}{\sqrt{M/C}} \]

\[ Z_C = \sqrt{M/C} \]

\[ H(j\omega) = \frac{\omega_0^2}{\omega_0^2 - \omega^2 + j\alpha \omega_0 \omega} = \frac{1}{(1 - \Omega^2) + j\alpha \Omega}, \]

\[ |H(j\omega)| = \frac{1}{\sqrt{(1 - \Omega^2)^2 + (\alpha \Omega)^2}} \]

\[ \theta_H = -\arctan \frac{\alpha \Omega}{1 - \Omega^2} \]

when \( \omega = \omega_0, \Omega = 1, |H| = 1/\alpha \) and \( \theta_H = -90^\circ \).
\[ Y(j\omega) = \frac{1}{M} \frac{j\omega}{(\omega_0^2 - \omega^2) + j\alpha\omega_0\omega} = \frac{1}{Z_C} \frac{j\Omega}{(1 - \Omega^2) + j\alpha\Omega}. \]

\[
|Y(j\omega)| = \frac{1}{Z_C} \frac{\Omega}{\sqrt{(1 - \Omega^2)^2 + (\alpha\Omega)^2}}
\]

\[ \theta_Y = \frac{\pi}{2} - \arctan \frac{\alpha\Omega}{1 - \Omega^2} \]

when \( \omega = \omega_0, \Omega = 1, |Y| = 1/(\alpha Z_C) = 1/R \) and \( \theta_H = 0^\circ \)
Figure 21: Dependence of the magnitude and phase of the input admittance system function $Y$ (solid curves) and pressure transfer ratio system function $H$ (dotted curves) on normalized frequency $\Omega = \omega/\omega_0$ for the acoustic circuit of Fig. 20. The magnitude of the admittance system function $Y$ has been multiplied by $Z_C$. The curves have been drawn for the case $\alpha = 0.1$. 
Figure 22: Dependence of the magnitude and phase of the input admittance system function $Y$ (solid curves) and pressure transfer ratio system function $H$ (dotted curves) on normalized frequency $\Omega = \omega/\omega_0$ for the acoustic circuit of Fig. 20. Log-log coordinates have been used. The magnitude of the admittance system function $Y$ has been multiplied by $Z_C$. The curves have been drawn for the case $\alpha = 0.1$. 
Figure 23: Circuit that has a single acoustic mass and a single acoustic compliance.

\[ \frac{U}{P_S} = \frac{1}{sM + 1/sC} = \frac{1}{M} \frac{s}{s^2 + \omega_0^2} \]

\[ \frac{P_O}{P_S} = \frac{1/sC}{sM + 1/sC} = \frac{\omega_0^2}{s^2 + \omega_0^2} \]
Figure 24: Circuit that has a single acoustic mass and a single acoustic compliance.

\[
\frac{U_M}{U_S} = \frac{1/sC}{sM + 1/sC} = \frac{\omega_0^2}{s^2 + \omega_0^2}
\]

\[
\frac{P_O}{U_S} = \frac{1}{\frac{1}{sM} + \frac{1}{sC}} = \frac{1}{C} \frac{s}{s^2 + \omega_0^2}
\]
\[ p_M = -p_C \]

\[ M \frac{du(t)}{dt} = -p_C(t) \]

\[ u(t) = C \frac{dp_C(t)}{dt}. \]

\[ MC \frac{d^2p_C(t)}{dt^2} + p_C(t) = 0, \]

\[ \frac{d^2p_C(t)}{dt^2} + \omega_0^2 p_C(t) = 0. \]

Figure 25: The circuits of Fig. 23 and 24 reduce to this two element circuit when the sources have zero intrinsic value.
\[ p_C(t) = P_C \cos (\omega t + \theta) \]
\[ \frac{dp_C}{dt} = -\omega_0 \sin (\omega t + \theta) \]
\[ \frac{d^2 p_C(t)}{dt^2} = -\omega_0^2 \cos (\omega t + \theta) = -\omega_0^2 p_C(t) \]
Figure 26: Example used to analyze power and energy in acoustic circuits.
\[ p(t) = p_R(t) + p_M(t) + p_C(t) \]
\[ w(t) = p(t)u_S(t) = p_R(t)u_S(t) + p_M(t)u_S(t) + p_C(t)u_S(t) \]
\[ \]
\[ p_R(t) = R_A u_S(t) \]
\[ p_M(t) = M_A \frac{du_S(t)}{dt} \]
\[ u_S(t) = C_A \frac{dp_C(t)}{dt} \]

\[ w(t) = R_A u_S^2(t) + M_A u_S(t) \frac{du_S(t)}{dt} + C_A p_C(t) \frac{dp_C(t)}{dt} \]
\[ w(t) = R_A u_S^2(t) + \frac{dE_M(t)}{dt} + \frac{dE_C(t)}{dt} \]
\[ w(t) = w_d(t) + \frac{d(E_M(t) + E_C(t))}{dt} \]
\[ u_S(t) = U_S \cos (\omega t + \phi) = \text{Re} [U_S e^{j\omega t}] \]

\[ p(t) = P \cos (\omega t + \theta) = \text{Re} [P e^{j\omega t}] \].

\[ w(t) = U_S P \cos (\omega t + \phi) \cos (\omega t + \theta). \]

\[ \cos x \cos y = \frac{1}{2} \cos (x - y) + \frac{1}{2} \cos (x + y) \]

\[ w(t) = \frac{1}{2} U_S P \cos (\theta - \phi) + \frac{1}{2} U_S P \cos (2\omega t + \theta + \phi) \]

\[ w(t) = \frac{1}{2} \text{Re} [PU_S^*] + \frac{1}{2} |P| |U_S| \cos (2\omega t + \theta + \phi) \]

\[ W_{av} = \frac{1}{2} \text{Re} [PU^*] \]
\[ W = \frac{1}{2} P U_s^* \]

\[ P = P_R + P_M + P_C \]

\[ P U_s^* = R_A |U_s|^2 + j\omega M_A |U_s|^2 - j\omega C_A |P_C|^2, \]

\[ W = \frac{1}{2} P U_s^* = \frac{1}{2} R_A |U_s|^2 + j2\omega \left( \frac{1}{4} M_A |U_s|^2 - \frac{1}{4} C_A |P_C|^2 \right) \]

\[ W = P_{av} + j2\omega (\langle E_M \rangle - \langle E_C \rangle). \]