chapter 2

large-scale variations of the average signal

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SYNOPSIS OF CHAPTER

The principal methods by which energy is transmitted to a mobile, namely, reflection and diffraction, are often indistinguishable; thus it is convenient to lump the losses together and call them scatter (or shadow losses). In Chapter 1 it was shown that this scattering gives rise to fields whose amplitudes are Rayleigh distributed in space. The assumption of the Rayleigh model led to very powerful results and will be used again in the studies of modulation and diversity in Chapters 4 and 5. In this chapter it is shown that while the "local statistics" may be Rayleigh, the "local mean" varies because of the terrain and the effects of other obstacles. Indeed, observations of the local mean indicate that it can be characterized statistically.

Transmission paths from mobiles to base stations can be extremely varied, ranging from the occasional direct line-of-sight path to severely shadowed paths from large terrain obstructions. Under certain conditions, which are usually much simplified, the path loss can be calculated exactly between two antennas. In Section 2.1, the transmission loss for line-of-sight antennas is given. Then transmission over a plane earth is discussed, and finally the diffraction losses due to simple geometric objects in the path are calculated. There are other factors such as rain, water vapor, and oxygen in the atmosphere that attenuate microwave transmission. These factors are discussed in the latter part of Section 2.1.

In Section 2.2, signal losses are considered where terrain effects are not a factor. It is seen that there is a considerable difference in scattering losses between a suburban area and a highly built-up urban area. Antenna heights, separation distances, and frequency affect propagation in both cases. Within a city, buildings tend to channel energy parallel to the
streets, strongly affecting the shadow losses.

Measurements over irregular terrain have also been made and are discussed in Section 2.3. The cases treated include rolling hills, large-scale slopes, land-sea paths, and transmission through tunnels and foliage.

A statistical representation of the local mean received signal is useful in estimating the coverage from a base station and for estimating cochannel interference. In Section 2.4 it is shown that the distribution of the local mean received signal is a log-normal distribution whose mean and variance depend on the environment. Section 2.5 concludes this chapter with predictions of path loss and estimates of the coverage surrounding a base station.

2.1 FACTORS AFFECTING TRANSMISSION

2.1.1 Free-Space Transmission Formula

The power received by an antenna separated from a radiating antenna is given by a simple formula, provided there are no objects in the region that absorb or reflect energy. This free-space transmission formula depends on the inverse square of the antenna separation distance, \( d \), and is given by

\[
P_0 = P_t \left( \frac{\lambda}{4\pi d} \right)^2 g_b g_m,
\]

(2.1-1)

where \( P_0 \) is the received power, \( P_t \) is the transmitted power, \( \lambda \) is the wavelength, \( g_b \) is the power gain of the base station antenna, and \( g_m \) is the power gain of the mobile station antenna.

Thus the received radiated power decreases 6 dB for each doubling of the distance. On first inspection, one might conclude that higher frequencies might be unsuitable for mobile communications because the transmission loss increases with the square of the frequency. However, this usually can be compensated for by increased antenna gain. In mobile communications, it is often desirable to have antennas whose patterns are omnidirectional in the azimuthal plane; thus the increase in gain is required in the elevation plane. In the limit of the higher microwave frequencies, this additional gain may become impractical to realize for effective communications between an elevated base station and a mobile. This problem is considered in more detail in Chapter 3. Since the most ideal mobile radio
path involves line-of-sight propagation through the atmosphere with antennas located near the earth, we will consider the effects of both in the following paragraphs.

2.1.2 Propagation Over a Plane Earth

Knowing the propagation characteristics over a smooth, conducting, flat earth provides a starting point for estimating the effects of propagation over actual paths. The complex analytical results for propagation over a plane earth derived by Norton 2-4 have been simplified by Bullington 5,6 by decomposing the solution of Norton into a set of waves consisting of direct, reflected, and surface waves. The formula relating the power transmitted to the power received following the approach of Bullington is

\[ P_r = P_t \left[ \frac{\lambda}{4\pi d} \right]^2 \frac{\sigma_0 \sigma_m}{e} \left| 1 + Re^{j\Delta} + (1 - R) Ae^{j\Delta} + \cdots \right|^2. \]  

(2.1-2)

Within the absolute value symbols, the first term (unity) represents the direct wave, the second term represents the reflected wave, the third term represents the surface wave, and the remaining terms represent the induction field and secondary effects of the ground.

The reflection coefficient, \( R \), of the ground depends on the angle of incidence, \( \theta \), the polarization of the wave, and the ground characteristics; it is given by

\[ R = \frac{\sin \theta - z}{\sin \theta + z} \]  

(2.1-3)

where

\[ z = \frac{\sqrt{\varepsilon_0 - \cos^2 \theta}}{\varepsilon_0} \]  

for vertical polarization,

\[ z = \sqrt{\varepsilon_0 - \cos^2 \theta} \]  

for horizontal polarization,

\[ \varepsilon_0 = \varepsilon - j60\sigma\lambda, \]

\[ \varepsilon = \text{the dielectric constant of the ground relative to unity in free space,} \]

\[ \sigma = \text{the conductivity of the earth in mhos per meter.} \]
The quantity $\Delta$ is the phase difference between the reflected and the direct paths between transmitting and receiving antennas, illustrated in Figure 2.1-1. Let $h_b$ and $h_m$ be the heights of the base and mobile antennas; then $\Delta$ is given by

$$\Delta = \frac{2\pi}{\lambda} \left[ \left( \frac{h_b + h_m}{d} \right)^2 + 1 \right]^{1/2}$$

$$- \frac{2\pi d}{\lambda} \left[ \left( \frac{h_b - h_m}{d} \right)^2 + 1 \right]^{1/2}.$$

(2.1-4)

For $d$ greater than $5h_b h_m$,

$$\Delta \approx \frac{4\pi h_b h_m}{\lambda d}.$$

(2.1-5)

Since the earth is not a perfect conductor, some energy is transmitted into the ground, setting up ground currents that distort the field distribution relative to what it would have been over a perfectly reflecting surface. The surface wave attenuation factor, $A$, depends on frequency, polarization, and the ground constants. An approximate expression for $A$ is given by

$$A \approx \frac{-1}{1 + j(2\pi d/\lambda)(\sin \theta + z)^2},$$

(2.1-6)

which is valid for $|A| < 0.1$. More accurate values are given by Norton.\(^3\)

Since the effect of this surface wave is only significant in a region a few wavelengths above the ground, this effect can be neglected in most applications of microwave mobile communications.

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Figure 2.1-1 Propagation paths over a plane earth.
It is of interest to note that in the limit of grazing angle of incidence the value of the reflection coefficient, $R$, approaches $-1$ independent of the polarization. For frequencies above 100 MHz and for an “average” earth (see Table 1) and for vertical polarization, $|R|$ exceeds 0.9 for angles less that 10° above the horizon. For horizontal polarization above 100 MHz, $|R|$ exceeds 0.5 for angles less than 5°, but must be of the order of a degree or less for $|R|$ to exceed 0.9.  

<table>
<thead>
<tr>
<th>Table 1 Typical Ground Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of Surface</strong></td>
</tr>
<tr>
<td>Poor ground</td>
</tr>
<tr>
<td>Average ground</td>
</tr>
<tr>
<td>Good ground</td>
</tr>
<tr>
<td>Sea water</td>
</tr>
<tr>
<td>Fresh water</td>
</tr>
</tbody>
</table>

Under the conditions where $R$ equals $-1$ and $A$ can be neglected, then (2.1-2) reduces to

$$P_r = 4P_0 \sin^2 \left( \frac{2\pi h_b h_m}{\lambda d} \right),$$

(2.1-7)

where $P_0$ is the expected power over a free space path. In most mobile radio applications, except very near the base station antenna, $\sin \frac{1}{2} \Delta \approx \frac{1}{2} \Delta$; thus the transmission loss over a plane earth is given by the approximation

$$P_r = P_t g_b g_m \left( \frac{h_b h_m}{d^2} \right),$$

(2.1-8)

yielding an inverse fourth-power relationship of received power with distance from the base station antenna.

The ground constants over the path of interest enter into both the calculations for line-of-sight and for diffraction attenuation. At microwave frequencies it is usually the dielectric constant, $\epsilon$, which has the dominant effect on propagation. Table 1 gives values of typical ground constants. Applying these values to the formulas for the reflection coefficient over a plane earth just derived, we find that for frequencies above 100 MHz the effect of the ground constants are slight.
2.1.3 Rough Surface Criterion

At the higher microwave frequencies the assumption of a plane earth may no longer be valid, due to surface irregularities. A measure of the surface "roughness" that provides an indication of the range of validity of Eq. (2.1-2) is given by the Rayleigh criterion, which is

$$C = \frac{4\pi\sigma}{\lambda},$$  \hspace{1cm} (2.1-9)

where $\sigma$ is the standard deviation of the surface irregularities relative to the mean height of the surface, $\lambda$ is the wavelength, and $\theta$ is the angle of incidence measured in radians from the horizontal. Experimental evidence shows that for $C < 0.1$ specular reflection results, and the surface may be considered smooth. Surfaces are considered "rough" for values of $C$ exceeding 10, and under these conditions the reflected wave is very small in amplitude. Bullington has found experimentally that most practical paths at microwave frequencies are relatively "rough" with reflection coefficients in the range of 0.2-0.4.

2.1.4 Refraction and Equivalent Earth's Radius

Because the index of refraction of the atmosphere is not constant, but decreases (except during unusual atmospheric conditions) with increasing height above the earth, electromagnetic waves are bent as they propagate. The mean variation in refractive index can be considered linear with a constant gradient $g$ of the form

$$n = n_0 + gh. \hspace{1cm} (2.1-10)$$

In a medium where there are abrupt changes in index of refraction, Descarte's law applies:

$$n(a+h)\cos\alpha = n_0a\cos\alpha_0, \hspace{1cm} (2.1-11)$$

where $\alpha$ and $\alpha_0$ are the angles at the discontinuity at height $h$, above the surface of the earth of radius $a$ (see Figure 2.1-2). Note if the atmosphere is uniform the equation for rectilinear propagation is

$$\left(1 + \frac{h}{a}\right)\cos\alpha = \cos\alpha_0. \hspace{1cm} (2.1-12)$$

When $n$ has a constant gradient the propagation is given approximately by

$$\left[1 + h\left(\frac{1}{a} + g\right)\right]\cos\alpha \approx \cos\alpha_0. \hspace{1cm} (2.1-13)$$
If we replace the earth's radius $a$ by a fictitious value $a'$, where

$$a' = \left(\frac{1}{a} + g\right)^{-1}, \quad (2.1-14)$$

we now have an expression in the same form as that for rectilinear propagation.

Since the index of refraction in the troposphere is very nearly unity, the $N$-unit has been defined for convenience,

$$N_s = (n - 1) \times 10^6, \quad (2.1-15)$$

where $n$ is the index of refraction in the atmosphere. Values of the minimum monthly mean value of $N_s$ throughout the world have been published. The most commonly used value for $N_s$ is 301. This gives a value for the effective earth's radius $a'$ which corresponds to four-thirds of the actual earth's radius. The empirical formula for $a'$ is given by

$$a' = 6370[1 - 0.04665 \exp(0.005577N_s)]^{-1} \text{ km}, \quad (2.1-16)$$

where 6370 km is used for the earth's radius.

2.1.5 Transmission over a Smooth Spherical Earth

At microwave frequencies, diffraction due to the earth severely limits the amount of energy that propagates beyond the horizon. Considerable work has been done in an attempt to predict the signal attenuation over transhorizon paths. Generally speaking, these predictions are semiempirical formulas which apply for frequencies below 1000 MHz. It is possible to obtain analytic expressions for the diffraction over a perfectly conducting sphere; however, the expressions are not simple relationships between the factors of frequency, conductivity of the earth, antenna height, and distance which govern the attenuation. A rigorous derivation of the diffraction over a spherical earth may be found in Chapter 8 of Jones. Estimations of the attenuation due to diffraction over a smooth earth are
particularly difficult in regions just beyond line-of-sight. Furthermore, surface roughness again seriously affects propagation. It is, of course, desirable to be able to estimate signal strengths beyond the horizon, particularly for cases where the same frequencies are being used at separated base stations. Bullington\cite{Bullington6} has reduced the involved analytic relationships for the propagation over a smooth spherical earth to various asymptotic forms. Figure 2.1-3 is a nomograph, accurate to ±2 dB, which was derived from his approximations. The distances $d_1$ and $d_2$ are the distances to the horizon, which can be written as

$$d_{1,2} = \sqrt{2a'h_{1,2}}$$  \hspace{1cm} (2.1-17)

where $h_{1,2}$ are the antenna heights and $a'$ is the effective earth's radius.

![Nomograph of signal attenuation from propagation over a smooth spherical earth.](image)

*Figure 2.1-3* Nomograph of signal attenuation from propagation over a smooth spherical earth.
2.1.6 Knife Edge Diffraction

Very often in the mobile radio environment a line-of-sight path to the base station is obscured by obstructions such as hills, trees, and buildings. When the shadowing is caused by a single object such as a hill, it is instructive to treat the object as a diffracting knife edge to estimate the amount of signal attenuation. The exact solution to the problem of diffraction over a knife edge is well known and is discussed in many textbooks (Ref. 11, for example).

Within the shadow region of the knife edge, the electric field strength $E$, can be represented as

$$\frac{E}{E_0} = A \exp (i\Delta),$$

(2.1-18)

where $E_0$ is the value of the electric field at the knife edge, $A$ is the amplitude, and $\Delta$ is the phase angle with respect to the direct path. The expressions for $A$ and $\Delta$ are obtained in terms of the Fresnel integrals:

$$A = \frac{S + 1/2}{\sqrt{2} \sin (\Delta + \pi/4)},$$

(2.1-19)

$$\Delta = \tan^{-1} \left( \frac{S + 1/2}{C + 1/2} \right) - \frac{\pi}{4},$$

(2.1-20)

where

$$C = \int_0^{h_0} \cos \left( \frac{\pi}{2} v^2 \right) dv,$$

(2.1-21)

$$S = \int_0^{h_0} \sin \left( \frac{\pi}{2} v^2 \right) dv,$$

(2.1-22)

and

$$h_0 = h \sqrt{\frac{2}{\lambda} \left( \frac{1}{d_1} + \frac{1}{d_2} \right)}.$$

(2.1-23)

For most microwave mobile radio applications several assumptions can be made to simplify the calculations. Consider an infinite completely absorbing (rough) half-plane that divides space into two parts as in Figure 2.1-4. When the distances $d_1$ and $d_2$ from the half-plane to the transmitting antenna and the receiving antenna are large compared to the height $h$, and
h itself is large compared with the wavelength, \( \lambda \), that is,

\[ d_1, d_2 \gg h \gg \lambda, \]  

(2.1-24)

then the diffracted power can be given by the expression

\[ \frac{P}{P_0} = \frac{1}{2\pi h_0^2}. \]  

(2.1-25)

This result can be considered independent of polarization as long as the conditions of Eq. (2.1-24) are met. In cases where the earth's curvature has an effect, there can be up to four paths. A simplified method of computing knife edge diffraction for such cases is treated by Anderson and Trolese.\(^{12}\)

Closer agreement with data over measured paths has been obtained by calculations that better describe the geometry of the diffracting obstacle.\(^{13-15}\)

![Figure 2.1-4](image)

**Figure 2.1-4** Geometry for propagation over a knife edge.

### 2.1.7 Effects of Rain and the Atmosphere

Microwave mobile radio signals are attenuated by the presence of rain, snow, and fog. Losses depend upon the frequency and upon the amounts of moisture in the path. At the higher microwave frequencies, frequency selective absorption results because of the presence of oxygen and water vapor in the atmosphere.\(^{16}\) The first peak in the absorption due to water vapor occurs around 24 GHz, while for oxygen the first peak occurs at about 60 GHz, as shown in Figure 2.1-5.

The attenuation due to rain has been studied experimentally\(^{17}\) by a number of workers. Figure 2.1-6 provides an estimate of the attenuating effect of rainfall as a function of frequency for several rainfall rates.\(^{16}\) It should be noted that very heavy rain showers are usually isolated and not large in extent. Nevertheless, at frequencies above 10 GHz the effects of rain cannot be neglected.
Figure 2.1-5  Signal attenuation from oxygen and water vapor in the atmosphere.

Figure 2.1-6  Signal attenuation for various rainfall rates.
2.1.8 Miscellaneous Effects

It is possible for microwave signals to propagate beyond the horizon by tropospheric scattering and by ducting due to diffraction. These phenomena could potentially cause interference problems; however, their effects usually can be neglected and the reader is referred to textbooks on the subject such as Ref. 8.

2.2 OBSERVED ATTENUATION ON MOBILE RADIO PATHS OVER SMOOTH TERRAIN

We will now depart from the realm of precise definitions and exact solutions to that of descriptive types of definitions and settle for results that are statistical at best. Signal attenuations over actual mobile radio paths result from a complicated dependence on the environment, and the formulas derived in the previous section apply only in special cases where the propagation paths can be clearly described. In this section we will concentrate our attention on propagation effects over relatively smooth terrain.

First, let us define the terrain to be "quasismooth" when the undulation of the ground about the average ground height is less than 20 meters. This definition would still allow the actual surface to be either "smooth" or "rough" according to the Rayleigh criterion. Furthermore, the undulations should be "gentle," that is, the distance between the peak and trough should be much larger than the height of the undulations. The average ground level should likewise remain constant to within 20 meters for distances of the order of kilometers.

The position of an antenna above the terrain affects transmission. A transmitting antenna, located 100 meters atop a mountain, certainly will propagate signals differently than an antenna located 100 meters above a flat plain. A definition of antenna height that accounts for this discrepancy is called for. Figure 2.2-1 illustrates a suitable definition for the base-station antenna height, $b$, which is the height of the antenna above the average ground level in the region of interest (usually considered to be greater than 3 km and less than 15 km).

Some sort of classification of the environment is also necessary since signal attenuation varies depending upon the type of objects that obstruct the path. Rather than attempt to precisely define many types of environments and then describe propagation characteristics for each we will restrict our definitions of environment to three types:

1. Open areas: areas where there are very few obstacles such as tall trees or buildings in the path, for instance, farm land or open fields.
2. **Suburban areas**: areas with houses, small buildings and trees, often near the mobile unit.

3. **Urban areas**: areas that are heavily built up with tall buildings and multistory residences.

![Diagram](image)

*Figure 2.2-1* Definition of effective base station antenna height.

Transition regions will undoubtedly occur between the classification types and special cases can be imagined that do not fit any of the three classifications. However, the results presented for these classifications should apply in a great many instances and provide a basis to estimate results for other environments. The special problems of signal attenuation through foliage and tunnels can be isolated, and each is treated briefly at the end of this section.

Extensive measurements of radio transmission loss over various terrains in unpopulated areas have been made in the frequency ranges from 20 to 10,000 MHz by workers at the Institute for Telecommunications Sciences, and these data have been tabulated in several ESSA reports. Mobile radio measurements in urban and suburban areas in the microwave region have been made by a number of workers. To date, the most extensive work in the field has been reported by Okumura et al. We will rely heavily upon their results to generate the prediction curves that are provided in the last portion of this chapter.

### 2.2.1 Field Strength Variation in Urban Areas

Since most vehicles are concentrated within large metropolitan centers, and likewise most mobile radio services will be initially provided in these areas, it is reasonable to use for a basis of comparison the median field strengths measured in quasismooth urban environments and then express suburban and open area measurements as departures from the baseline
environment. Within the urban environment the received field strength is found to vary with the base station and mobile antenna heights, transmitting frequency, the distance from the transmitter, and the width and orientation of the streets. The median field strengths in a quasismooth urban area show a relatively continuous change with frequency, antenna height, and distance, while other effects appear less simply related.

With the imprecise definition of a quasismooth urban area and the variabilities of building heights and street widths that occur not only among cities but within them, one should be cautious about the range of applicability. However, as we proceed we shall see that there are many examples that indicate that there is consistency in characterizing propagation in complex environments. First, to appreciate the complexity of the situation let us examine in some detail the field strength measurements made in an urban area, and then we shall proceed to develop methods for statistically predicting signal coverage.

2.2.2 An Example of Signal Propagation

The following example is based upon measurements made by Black and Reudink at 836 MHz in Philadelphia, Pennsylvania. Continuous field strength recordings were made of the signals received by a mobile unit as it moved about in the downtown area. The transmitter was located on the tallest building in the city at a height of 500 ft, and the areas covered on the test runs in the mobile unit were in the southwest quadrant of the city. Two sections studied in detail consisted of a smaller area extending about 1 mile west and 1 mile south of the transmitting location, and a larger area extending 1 mile west and 3 miles south, as indicated in Figure 2.2-2.

A one-quarter wavelength vertical whip antenna was mounted on the ground plane on the roof of the test vehicle and was used to detect the signals from the transmitter. The height above ground of the receiving antenna was approximately 3 meters. A multichannel FM magnetic tape recorder was used to record the signal strength, voice commentary, and a marker tone that was geared to one of the wheels of the vehicle so that locations could be determined accurately.

Figure 2.2-3 is an example of "local mean" signal level taken from recordings that were transferred from magnetic tape to pen recorder. The upper illustration shows a portion of Pine Street where the signal level was very low due to the shadowing effects of tall buildings. The gradual improvement going from VanPelt Street to 20th Street can be seen, and the calibration scale to the left gives an indication of the magnitude of this gradual change. The lower portion of the figure illustrates a peaking up of the signal level at street intersections such as Chestnut, Ranstead, and Ludlow Streets as the vehicle was traveling along 18th Street.
Figure 2.2-2  Aerial view of Philadelphia, Pennsylvania.
Figure 2.2-3 Typical average signal level recordings.

Figure 2.2-4 Approximate region covered in smaller area model.
The approximate region covered in the smaller test area as viewed from the base-station transmitter is seen in Figure 2.2-4. There are several buildings greater than ten stories in height within this area. Occasionally one can see directly to the street below, but most often the line-of-sight path was obscured. A cardboard model was constructed to correspond in size with the sections of the street map to give a visual picture of the variation in received signal strength within the area under study. The signal level for each street was plotted on a perpendicular strip of cardboard. The strips were then assembled with other sections to form a three-dimensional pattern. Figure 2.2-5 is a view of the field strength model as seen from behind the transmitter, which is indicated by the black vertical strip. This view shows the signal level variation along streets looking from the transmitted location in a southwest direction. Some items of interest are (1) the generally good signal level along the wide streets such as Market and Broad Street, (2) the variation of 20 dB or more along such streets near the transmitter caused by shadowing effects of the tall buildings near the transmitter, and (3) the general rise in signal level near the river as the signal path approached line-of-sight conditions. Figure 2.2-6 is a view from the south of the area looking in a northeast direction. There is a region of low signal extending out from the transmitter in a southwest direction, which can be seen clearly; the peaking up of the signal at street intersections is also evident at several points in this view.

*Figure 2.2-5*  Model of fine detail test area.
Figure 2.2-6  Model of fine detail test area.

Figure 2.2-7  Approximate region covered in larger area model.
The approximate area covered in the test runs from the larger area study is shown in the photograph in Figure 2.2-7, which is a view looking approximately southwest from the base-station transmitter. Figure 2.2-8 is a view of the field strength model from East Broad Street looking from the transmitter in a westerly direction. Figure 2.2-9 is a view of the model from the western side looking toward the transmitter in an easterly direction. The depressed signal area near 21st and Spruce Streets is seen to be small in comparison with the whole area.

There is considerably less severe variation of signal strength on the larger area model than on the smaller area model. Much more of this area is now out of the central portion of the city and the buildings are more uniform. As we shall see later, data such as these, when analyzed, have several consistent characteristics that allow statistical predictions of signal coverage.

2.2.3 Distance Dependence

One of the fundamental problems in the study of radio propagation is to describe the manner in which the signal strength attenuates as the receiving unit moves away from a transmitting base station. Obviously the signal level will fluctuate markedly (even when the Rayleigh fading is averaged out) since building heights, street widths, and terrain features are not constant. However, if we consider for a moment the behavior of the median values of the received signals, we find that there is a general trend...
for the signal levels to decrease more rapidly the further the mobile is separated from the base station. Figure 2.2-10 is a plot of the received signal power versus distance as measured by independent workers in three different cities—New York, \(^{23}\) Philadelphia, \(^{22}\) and Tokyo. \(^{27}\) All the measurements were made at approximately 900 MHz from relatively high base-station locations. This remarkably consistent trend of both the falloff of the median signal value with distance and the excess attenuation relative to free space leads one to hope that signal strength parameters in different cities will likewise exhibit consistent traits.

The rate-of-signal decreases with distance does not appear to change significantly with increasing antenna height. However, raising the base-station antenna does tend to decrease the attenuation relative to free space. Figure 2.2-11 and 2.2-12 show measurements of these effects at 922 and at 1920 MHz by Okumura \(^{27}\) in Tokyo. The distance dependence relative to free space, based on the measurements of Okumura, is shown in Figure 2.2-13 for an antenna height of 140 meters for frequencies of 453, 922, 1430, and 920 MHz. Up to distances of about 15 km the signal strength relative to free space falls off at a rate approximately proportional to the distance from the base-station antenna. At large separation distances, the signal level decreases at a much more rapid rate.

The relationship between distance dependence and antenna height (again based upon the measurements of Okumura) is shown in Figure 2.2-14. For antenna separation distances between 1 and 15 km the attenuation of median signal power with distance changes from nearly an inverse
Figure 2.2-10  Examples of transmission loss with distance.

Figure 2.2-11  Median field strengths in an urban area for various antenna heights.
Figure 2.2-12  Median field strengths in an urban area for various antenna heights.

Figure 2.2-13  Distance dependence of median field strength attenuation (relative to free space) in urban area.
fourth power decrease for very low base-station antenna heights to a rate only slightly faster than the free-space decrease for extremely high base-station antennas. For antenna separation distances greater than 40 km, the signal attenuation is very rapid.

![Figure 2.2-14](image)

**Figure 2.2-14** Distance dependence of median field strength in an urban area ($P_{rd}d^{-n}$).

### 2.2.4 Frequency Dependence

Signal attenuation increases in urban areas as the frequency increases. For a fixed antenna height, the signal attenuation as a function of distance can be expressed in terms of $n$ ($P_r \propto f^{-n}$, where $P_r$ is the median received signal power) for varying frequencies. Figure 2.2-15 shows that $n$ is roughly constant for distances under 10 km from the base station. As the separation increases, the decrease in signal strength with frequency becomes more rapid.

![Figure 2.2-15](image)

**Figure 2.2-15** Frequency dependence of median field strength in an urban area ($P_r af^{-n}$).
Figure 2.2-16 is a prediction curve derived by Okumura for the basic median signal attenuation relative to free space in a quasismooth urban area as it varies with both distance and frequency. These curves provide the starting point for predicting signal attenuation as discussed in Section 2.5. The curves assume a base-station antenna height of 200 meters and a mobile antenna height of 3 meters. Adjustments to these basic curves for different base station antenna heights and mobile antenna heights are considered in the paragraphs that follow.

![Prediction curve for basic median attenuation relative to free space in urban area over quasi-smooth terrain, referred to \( h_b = 200 \text{ m} \) and \( h_m = 3 \text{ m} \).](image)

**Figure 2.2-16** Prediction curve for basic median attenuation relative to free space in urban area over quasi-smooth terrain, referred to \( h_b = 200 \text{ m} \), \( h_m = 3 \text{ m} \).

### 2.2.5 Antenna Height Dependence

The formulas derived in Section 2.1 for transmission over a plane earth made no unique distinction between the effects of raising (or lowering) either the base-station or mobile-station antenna. Equation (2.1-8) predicts a 6 dB gain in received power for a doubling of the height of either antenna. In typical real-life situations the mobile antenna is likely to be
buried within the confines of its surroundings while the base-station antenna will be elevated to some extent above local obstacles. The effect of a change of elevation is different in the two instances, and thus we shall treat the two cases separately.

**Effect of Base-Station Antenna Height**

Okumura\(^{27}\) has found that the variation of received field strength with distance and antenna height remains essentially the same for all frequencies in the range from 200 to 2000 MHz. For antenna separation distances less than 10 km the received power varies very nearly proportional to the square of the base-station antenna height (6 dB per octave). For very high base-station antennas and for large separation distances (greater than 30 km), the received power tends to be proportional to the cube of the height of the base station antenna (9 dB per octave). Figure 2.2-17 is a set of prediction curves that give the change in received power (often called the height-gain factor) realized by varying the base-station antenna height. The curves are plotted for various antenna separation distances and predict the median received power relative to a 200-meter base-station antenna and a 3-meter mobile antenna. They may be used for frequencies in the range from 200 to 2000 MHz.

![Figure 2.2-17](image-url)

*Figure 2.2-17* Prediction curves for base station height-gain factor referred to \( h_b = 200 \) meters.
Effect of Mobile Antenna Height

For obvious reasons mobile antenna heights are generally limited to no more than 4 meters. For a large range of frequencies and for several base station antenna heights Okumura observed a height-gain advantage of 3 dB for a 3-meter-high mobile antenna compared to a 1.5-meter-high mobile antenna. For special cases where antenna height can be above 5 meters, the height-gain factor depends upon the frequency and the environment. In a medium sized city where the transmitting frequency is 2000 MHz, the height-gain factor may be as much as 14 dB per octave, while for a very large city and a transmitting frequency under 1000 MHz the height-gain factor may be as little as 4 dB per octave for antennas above 5 meters. Prediction curves for the vehicle height-gain factor in urban areas are given in Figure 2.2-18.

![Figure 2.2-18](image)

*Figure 2.2-18* Prediction curve for vehicular height gain factor referred to $h_m = 3$ meters.
2.2.6 Correction Factor for Suburban and Open Areas

Suburban areas are generally characterized by lower buildings and generally less congestion of obstacles than in cities. Consequently, one should expect that radio signals would propagate better in such environments. Okumura has found that there is practically no change in the difference between urban and suburban median attenuation (suburban correction factor) with changes in base-station antenna height or with separation distances between the base and mobile antennas. The signal strength in suburban areas is weakly dependent on frequency and increases to some extent at the higher frequencies. A plot of the suburban correction factor is shown by the solid curve in Figure 2.2-19 for frequencies in the range of 100 to 3000 MHz. Recent data reported by Reudink\(^{24}\) shows a 10 dB difference between urban and suburban values of the median received signal strength at a frequency of 11,200 MHz, slightly less than that predicted by Okumura in Figure 2.2-19.

![Figure 2.2-19 Prediction curves for suburban and open area correction factor, \(K_{s0}\).](image)
Open areas that occur rather infrequently tend to have significantly better propagation paths than urban and suburban areas, and typical received signal strengths run nearly 20 dB greater for the same antenna height and separation distances. The upper curve shown in Figure 2.2-19 provides a correction in dB that may be added directly to the prediction values for the urban case. Rural areas or areas with only slightly built-up sections have a median signal strength somewhere between the two curves.

### 2.2.7 Effects of Street Orientation

It has been observed that radio signals in urban areas tend to be channeled by the buildings so that the strongest paths are not necessarily the direct paths diffracted over the edge of nearby obstructing buildings, but are found to be from directions parallel to the streets. Streets that run radially or approximately radially from the base station are most strongly affected by this channeling phenomena. This causes the median received signal strength to vary by as much as 20 dB at locations near the transmitter, as shown in the signal strength model in Figure 2.2-16. Figure 2.2-20 is a sketch that indicates the way in which signal strengths may vary in an urban area because of street orientation. The density of arrows represents the relative signal strength along the various streets.

The distribution of the signal paths in the horizontal plane as seen from the mobile vehicle in an urban area is strongly affected by the street and

![Figure 2.2-20](image-url)  
*Figure 2.2-20*  Idealized signal flow when channeling effects predominate.
building layout. Tests by Reudink in New York City indicate that the signals arriving parallel to the direction of the street are typically 10–20 dB higher than waves arriving at other angles. These tests were carried out at 11.2 GHz by scanning with a highly directive antenna (beamwidth of 5°) at various locations in the city. Three examples of received signal strength versus angle are shown in Figure 2.2-21. In one case (on Madison Avenue) the direction in which the signal strength was strongest was nearly in the opposite direction to the base-station antenna.

2.2.8 Effects of Foliage

There are a great many factors that affect propagation behind obstacles such as a grove of trees. Precise estimates of attenuation are difficult because tree heights are not uniform; also, the type, shape, density, and distribution of the trees influence the propagation. In addition, the density of the foliage depends on the season of the year. However, some success has been obtained by treating trees as diffracting obstacles with an average effective height.

An experimental study of propagation behind a grove of live-oak and hackberry trees in Texas for several frequencies has been reported by Lagrone. Height-gain measurements were made at several fixed distances behind the grove of trees for horizontally polarized waves. The measured results were compared to theoretical curves obtained assuming propagation over a smooth spherical earth and by assuming two-path diffraction over an ideal knife edge. At 82 MHz the trees were found to be fairly transparent, attenuating the signal approximately 1.6 dB per 100 ft. At a frequency of 210 MHz the absorption was found to be approximately 2.4 dB per 100 ft. At large distances of the order of 100 meters from the trees, whose heights were approximately 10 meters, the measured data fit the knife edge predictions fairly well, as seen in Figure 2.2-22. Here the measured signal strength at a frequency of 210 MHz and at a distance of 215 ft from the diffracting trees is compared to theoretical knife edge diffraction, assuming an effective height of the trees of 25 ft. At closer distances the agreement with theory is not as good. This is probably because the heights and distances from the trees cannot be clearly defined.

At frequencies from about 0.5 to 3 GHz and for distances greater than about five times the tree height, the measurements were in good agreement with the theoretical predictions of diffraction over an ideal knife edge, assuming distances and heights the same as those in the measurements. Figure 2.2-23 is a curve similar to Figure 2.2-22 but at a frequency of 2950 MHz. The measured data compared with the theoretical curves for a smooth spherical earth and with knife edge diffraction curves show better agreement with the knife edge diffraction theory. At shorter distances
Figure 2.2-21  Examples of directions of signal arrivals.
some propagation takes place through the trees, and this acts to reduce the effective height of the diffracting edge and at the same time increases the apparent distance from the diffracting edge to the antenna.

Recent measurements at 836 MHz and 11.2 GHz were made by Reudink and Wazowicz, they compare the signal strength measured on the same streets in summer and in winter in suburban Holmdel, New Jersey.
Figures 2.2-24 and 2.2-25 show signal strengths from data taken on a road that runs approximately perpendicular to a radial line 2 miles distant from a 400-ft base-station antenna site. The relative received signals are shown at a frequency of 836 MHz in both summer and winter in Figure 2.2-24. The corresponding data at 11.2 GHz are plotted in Figure 2.2-25. The curves for predicted values were derived from the knife edge diffraction formula and show reasonably good agreement. At the UHF frequency the average received signal strength in the summer when the trees were in full leaf was roughly 10 dB lower than for the corresponding locations in later winter. At X-band frequencies the losses during the summer appear to be greater in the areas where the signal levels were previously low.

### 2.2.9 Signal Attenuation in Tunnels

It is well known that frequencies in the VHF region commonly used for mobile communications are severely attenuated in tunnel structures. Only by using special antennas are these frequencies usable in long (over 1000 ft) tunnels. However, at microwave frequencies tunnels are effective guiding or channeling mechanisms and can offer significant improvement over VHF for communications.

A test was performed in the center tube of the Lincoln Tunnel, 8000 ft long, which connects midtown Manhattan to New Jersey under the Hudson River. The inside of the tunnel is roughly rectangular in cross section with a height of 13.5 ft and a width of 25 ft. Seven test frequencies roughly an octave apart were used to make signal attenuation measurements at the following frequencies: 153, 300, 600, 980, 2400, 6000, and 11,215 MHz. The transmitters were stationed 1000 ft inside the western portal in order to keep the test situation as simple as possible. This location cleared an initial curve at the entrance and allowed a line-of-sight path of nearly 2000 ft before an elevation change cut off the view. Beyond this point nearly another mile of tunnel remained before the eastern exit was reached.

The average loss of signal strength in dB against the antenna separation for the seven frequencies is plotted in Figure 2.2-26. For convenience in plotting the data, an arbitrary reference level of 0 dB at 1000 ft antenna separation was chosen. It is worth noting that the 153- and 300-MHz attenuation rates are nearly straight lines, implying that the signal attenuation has an exponential relationship to the separation. At 153 MHz the loss is extremely high (in excess of 40 dB per 1000 ft), where at 300 MHz the rate of attenuation is of the order of 20 dB per 1000 ft. At higher frequencies a simple exponential attenuation rate is not evident. In Figure 2.2-27 the data have been replotted on a logarithmic distance scale. Signal attenuations that depend upon distance raised to some power appear as
**Figure 2.2-24** Signal propagation at nearly a constant radius from an elevated base station.

**Figure 2.2-25** Signal propagation at nearly a constant radius from an elevated base station.
straight lines in this case. For the major portion of the length of the tunnel
the received signal level at 900 MHz has an inverse fourth-power depen­
dence upon the antenna separation, while at 2400 MHz the loss has an
inverse square dependence. At frequencies above 2400 MHz, dependence
of the signal strength with antenna separation is less than the free-space
path loss (throughout most of the length of the tunnel). Roughly, the
attenuation rates appear to be only 2–4 dB per 1000 ft for frequencies in
the 2400–11,000 MHz range.

Figure 2.2-26 Signal loss versus antenna separation for seven frequencies.

2.3 EFFECTS OF IRREGULAR TERRAIN

The traditional approach of predicting attenuation from propagation
over irregular terrain has been to approximate the problem to one that is
solvable in closed form. This is usually done by solving problems dealing
with smooth regular boundaries such as planes or cylinders. Several
workers have published approximations to the exact formulas for propaga-
tion over various obstacles. In Section 2.1 examples were given of the theoretical signal attenuation due to the earth’s curvature and due to an ideal absorbing knife edge. Improved agreement with measurements for cases such as the propagation over an isolated mountain ridge are obtained by more accurately describing the obstacle in terms of more realistic geometries.

![Signal loss versus log of antenna separation for seven frequencies.](image)

**Figure 2.2-27** Signal loss versus log of antenna separation for seven frequencies.

### 2.3.1 Modeling Propagation by Frequency Scaling

To some extent propagation over more complicated obstacles can be determined by constructing models and performing laboratory experiments at optical frequencies or frequencies in the millimeter wavelength region. Using a helium–neon gas laser operating at a wavelength of 632.8 nm with cylindrical diverging lens, Hacking constructed a model transmitter and used a narrow-slit aperture to model a receiver. With this arrangement the wavelength scaling to UHF is approximately $10^6$ to 1. Model hills could thus be constructed with workable dimensions. First obtaining agreement with theory by testing the system on smooth diffracting cylinders, Hacking has investigated more complicated obstacles such as double-hump or flat-top slabs. In addition to smooth objects, terrain
roughness was simulated by wrapping abrasive papers around the cylinders. In this manner surface roughness whose rms deviation ranged from $2\lambda$ to $18\lambda$ were obtained. Figure 2.3-1 is a plot of the results obtained by this method. The solid curve is the theoretical calculation of the diffraction loss over a perfectly conducting smooth cylinder. Examples of the excess loss due to surface roughness are shown in the curves labeled A through D. In attempting to apply the rough surface to a real-life example, curve B would correspond to the field strength received at UHF in the shadows of hills on which there are irregular distributions of houses and trees. (Assume, for example, that there are two houses whose dimensions are $30 \times 20 \times 30$ ft and two trees whose dimensions are $15 \times 15 \times 30$ ft per acre. Then the rms surface height is calculated to be 6 ft.)

![Graph of diffraction loss](image-url)

**Figure 2.3-1** Examples of modelling propagation of rough objects.
Other ways of estimating signal attenuation over irregular terrain are empirical formulas derived from measurements. We shall briefly mention a computer method that predicts path loss in good agreement with experiment, and then present some prediction curves that provide quick but somewhat less accurate estimates of signal attenuation.

2.3.2 A Computer Method for Predicting Attenuation

A computer program has been published by Longley and Rice\(^9\) that predicts the long-term median radio transmission loss over irregular terrain. The method predicts median values of attenuation relative to the transmission loss free space and requires the following: the transmission frequency, the antenna separation, the height of the transmitting and receiving antennas, the mean surface refractivity, the conductivity and dielectric constant of the earth, polarization, and a description of the terrain. This program was based on thousands of measurements and compares well with measured data\(^{52}\) over the following ranges:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>20 MHz to 40 GHz</td>
</tr>
<tr>
<td>Antenna height</td>
<td>0.5 to 3000 meters</td>
</tr>
<tr>
<td>Separation distance</td>
<td>1 to 2000 kilometers</td>
</tr>
<tr>
<td>Surface refractivity</td>
<td>250 to 450 (N) units</td>
</tr>
</tbody>
</table>

The critical parameters necessary in any prediction of path loss are those which characterize the terrain. The "interdecile" (see illustration in Figure 2.3-2) range, \(\Delta h(d)\), of terrain heights above and below a straight line is a parameter often used in prediction formulas and is calculated at fixed distances, \(d\), along the path. Longley and Rice have found that the values of \(\Delta h(d)\) increase with path length to an asymptotic value \(\Delta h\) according to the following formula:

\[
\Delta h(d) = \Delta h [1 - 0.8 \exp(-0.02d)],
\]

(2.3-1)

where \(\Delta h(d)\) and \(\Delta h\) are in meters and the distance \(d\) is in kilometers. For a particular path where profiles are available, \(\Delta h(d)\) can be calculated precisely. In other cases or for area predictions, estimates of \(\Delta h\) are given in Table 2.
The computer method of Longley and Rice provides both point-to-point and area predictions that agree well with the experiment, but a description of the calculation of the many parameters used in their method would be rather lengthy. The reader is referred to their work for precise calculations. We shall adopt a somewhat less accurate method in which we obtain correction factors to our basic median curves (Figure 2.2-16) for various terrain effects.

Table 2  \textit{Estimates of} $\Delta h$

<table>
<thead>
<tr>
<th>Type of Terrain</th>
<th>$\Delta h$ (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water or very smooth plains</td>
<td>0–5</td>
</tr>
<tr>
<td>Smooth plains</td>
<td>5–20</td>
</tr>
<tr>
<td>Slightly rolling plains</td>
<td>20–40</td>
</tr>
<tr>
<td>Rolling plains</td>
<td>40–80</td>
</tr>
<tr>
<td>Hills</td>
<td>80–150</td>
</tr>
<tr>
<td>Mountains</td>
<td>150–300</td>
</tr>
<tr>
<td>Rugged mountains</td>
<td>300–700</td>
</tr>
<tr>
<td>Extremely rugged mountains</td>
<td>$&gt;700$</td>
</tr>
</tbody>
</table>
2.3.3 Predictions of Propagation by Correction Factors: Undulating, Sloping, and Land-Sea Terrain

An approximate prediction curve for undulating terrain is given in Figure 2.3-2, which is based on work reported by the CCIR\(^\text{16}\) and Okumura\(^\text{27}\). This estimates the correction factor to the basic median attenuation curves derived previously in Section 2.1 for quasismooth urban terrain. More exact predictions would probably have some dependence on frequency and on antenna separation distance. If the location of the vehicle is known to be near the top of the undulation, the correction factor in Figure 2.3-2 can be ignored. On the other hand, if the location is near the bottom of the undulation, the attenuation is higher and is indicated on the lower curve on Figure 2.3-2.

![Figure 2.3-2 Definition of average angle of general terrain slope.](image)

In cases where the median height of the ground is gently sloping for distances of the order of 5 km, a correction factor may be applied. Let us define the average slope $\theta_m$ measured in milliradians as illustrated in Figure 2.3-3. Depending upon the antenna separation distance $d$, the terrain slope correction factor, $K_{sp}$, is given in Figure 2.3-4 in terms of the average slope $\theta_m$. (It should be noted that these curves are based on rather scant data and should be considered as estimates applying in the frequency range of 450–900 MHz.)

Usually on propagation paths where there is an expanse of water between the transmitting and receiving stations the received signal strength
tends to be higher than for cases where the path is only over land. The change in signal strength depends on the antenna separation distance and whether the water lies closer to the mobile receiver or the base-station transmitter, or somewhere in between. Let us define a ratio, $\beta$, which represents the fraction of the path that consists of propagation over water. Figure 2.3-5 illustrates two path geometries and the definition of $\beta$ in each case. It has been observed that when the latter portion of the path from the

Figure 2.3-4 Measured value and prediction curves for “slope terrain correction factor.”

Figure 2.3-5 Prediction curves for land-sea correction factor.
base-station antenna to the mobile antenna is over water, the signal strength is typically 3 dB higher than for cases where the latter portion of the base mobile path is over land. Prediction curves for mixed land-sea paths have been obtained experimentally by Okumura, as shown in Figure 2.3-5, which provides correction factors in terms of the percentage of the path over water.

2.4 STATISTICAL DISTRIBUTION OF THE LOCAL MEAN SIGNAL

Thus far we have obtained results based primarily on experimental evidence that has provided us with the behavior of median signal levels obtained by averaging received signals over a distance of 10–20 meters. Smooth curves were obtained relating the variation of the median received signal with distance, base-station antenna height, and frequency in urban, suburban, and rural areas. Consistent but less accurate predictions were found for dependence on street orientation, isolated ridges, rolling hills, and land-sea paths. Considerably fewer data are available to describe the fluctuations of the received signal about the median value. The dependence of the signal distribution upon the parameters mentioned above requires a good deal more investigation before definitive results are available.

![Figure 2.4-1](image)

*Figure 2.4-1* Histogram of excess path loss in New Providence, N.J.
One consistent result that has been observed is that the distribution of the received signals at fixed base and mobile antenna heights, frequency, and separation distance from the base station within the same environment class (urban, for example) have very nearly a normal distribution when the distribution is plotted for the received signal measured in decibels. Such a probability distribution is often referred to as log-normal. Also the excess path loss, that is, the difference (in decibels) between the computed value of the received signal strength in free space (Eq. 2.1-1) and the actual measured value of the local mean received signal has been observed to be log-normally distributed. Figures 2.4-1 and 2.4-2 are examples of histograms of excess path loss measured at 11.2 GHz in a suburban area of New York City.
Large-Scale Variations of the Average Signal

New Jersey and in New York City.\textsuperscript{24} The distributions of these two histograms are plotted in Figure 2.4-3. The median values of the received signal in the urban case is about 10 dB lower than for the suburban case. Both sets of data appear to fit straight lines rather well, corresponding to a standard deviation of 10 dB. The data were also sorted into various range slots and their corresponding distributions were calculated. For the suburban data no significant changes were observed. In the urban case, however, the standard deviation of the excess path loss was found to decrease to about 8 dB for locations less than 1 mile from the base-station transmitter and to increase to 12 dB for locations greater than 1 mile from the base station.\textsuperscript{24}

![Figure 2.4-3](image)

\textbf{Figure 2.4-3} Distributions of excess path loss in New Providence, N.J. and New York City.

The distribution of excess path loss was also calculated for the signal strength models constructed from the measurements of the local mean signal received at 836 MHz in Philadelphia discussed in Section 2.2. Figure 2.4-4 is a plot of the distributions of the received signal levels for the models shown in Figures 2.2-5 and 2.2-8. In Philadelphia Black and Reudink\textsuperscript{22} noted an increase in $\sigma$, the standard deviation, for distances close to the base station (see Figure 2.4-4). However, measured data in
New York showed the opposite effect, namely, that the value of $\sigma$ increased with distance. A partial explanation for these seemingly contradictory results is that in the Philadelphia case, for distances greater than 1 mile from the base station, the average building height was about 10–20 meters, tending to produce a low value of $\sigma$. On the other hand, in New York City at distances greater than 1 mile from the base station there are still many very tall buildings, some well over 100 meters, whose presence would tend to increase $\sigma$.

\[ \text{LARGER AREA MODEL} \]

\[ \text{SMALLER AREA MODEL} \]

Figure 2.4-4 Distributions of excess path loss in Philadelphia.

Okumura has measured the standard deviation of the median field strength variations in Tokyo and has found that the mean values of the standard deviation are not strongly dependent upon the base-station...
Large-Scale Variations of the Average Signal

antenna height or antenna separation distance but do have a slight dependence with frequency. In these measurements base-station antenna heights ranged from 140 to 820 meters. For lower antenna heights where there are obstacles in the path comparable in height to the base-station antenna height, one would suspect that the standard deviation would increase somewhat. Measurements for base-station antenna heights ranging from 15 to 25 meters were made in the Philadelphia area by Ott. His results show a standard deviation that decreased very slightly with distance from a nominal value of 8.2 dB at 1.5 km to 7 dB at 15 km.

Figure 2.4-5 is a prediction curve for the standard deviation, $\sigma$, of the log-normal distribution that describes the variation of the median signal strength values in suburban areas as given by Okumura. The data spreads at 850 MHz are from the data of Black and Ott; at 11.2 GHz the data are from Reudink. The data points at 127 and 510 MHz are from the work of Egli.

![Figure 2.4-5](image)

**Figure 2.4-5** Prediction curves for standard deviation of median field strength variation in urban, suburban, and rolling hilly terrain.

2.5 PREDICTION OF FIELD STRENGTH

2.5.1 Prediction of Median Signal Strength

In order to predict the median power received by a mobile unit from a base-station antenna in a basic urban environment we may use the
following equation (all quantities in decibels):

$$P_p = P_o - A_m(f, d) + H_b(h_b, d) + H_m(h_m, f),$$ (2.5-1)

where

- $P_p$ represents the value of the predicted received power,
- $P_o$ is the power received for free-space transmission (Eq. 2.1-1)
- $A_m(f, d)$ is the median attenuation relative to free space in an urban area where the effective base-station antenna height, $h_b$, is 200 meters and the vehicle antenna height, $h_m$, is 3 meters. These values are expressed as a function of distance and frequency and can be obtained from the curves of Figure 2.2-16.
- $H_b(h_b, d)$ is the base-station height-gain factor expressed in decibels relative to a 200-meter-high base-station antenna in an urban area. This function is dependent upon distance and has been plotted in Figure 2.2-17.
- $H_m(h_m, f)$ is the vehicle station height-gain factor expressed in decibels relative to a 3-meter-high vehicle-station antenna in an urban area. This factor is dependent upon frequency and has been plotted in Figure 2.2-18.

If the particular propagation path happens to be over a different environment type or to involve terrain that is not "quasismooth," we may amend our prediction formula for $P_p$ by adding one or more of the correction factors that were described in earlier portions of the chapter. Thus, the "corrected" predicted received power $P_c$ is

$$P_c = P_p + K_{so} + K_{sp} + K_{ls} + K_{ter},$$ (2.5-2)

where

- $K_{so}$ is the "correction factor for suburban and open terrain," which is plotted in Figure 2.2-19.
- $K_{sp}$ is the "correction factor for sloping terrain," which is obtained from the curve in Figure 2.3-4.
- $K_{ls}$ is the "land-sea correction factor," which provides a correction to the signal attenuation when there is an expanse of water in the propagation path. A description of this correction factor and a prediction curve are given in Figure 2.3-5.
- $K_{ter}$ is the "correction factor for rolling hilly terrain," which was discussed in Section 2.3 and may be obtained from Figure 2.3-2.

In addition to these correction factors, there are other factors such as isolated mountain ridges, street orientation relative to the base station, the presence or absence of foliage, effects of the atmosphere, and in the case of
undulating terrain, the position of the vehicle relative to the median height. These additional effects together with the fact that the correction factors and indeed the basic transmission factors $A_m$, $H_b$, and $H_m$ are average values based upon empirical data, should indicate that discrepancies between measured and predicted values are still possible. It is reassuring to point out, however, that these prediction curves, which are essentially the same as those of Okumura, have been compared to measured data with a great deal of success. This was done over a variety of environments, antenna heights, and separation and for a number of frequencies.

2.5.2 Determination of Signal Coverage in a Small Area

Let us assume that the local mean signal strength in an area at a fixed radius from a particular base-station antenna is log-normally distributed. Let the local mean (that is, the signal strength averaged over the Rayleigh fading) in decibels be expressed by the normal random variable $x$ with mean $\bar{x}$ (measured in dBm*, for example) and standard deviation $\sigma$ (decibels). To avoid confusion, recall that $\bar{x}$ is the median value found previously from Eq. (2.5-2). As we have seen, $\bar{x}$ depends upon the distance $(r)$ from the base station as well as several other parameters. Let $x_0$ be the receiver “threshold.” We shall determine the fraction of the locations (at $r = R$) wherein a mobile would experience a received signal above “threshold.” The “threshold” value chosen need not be the receiver noise threshold, but may be any value that provides an acceptable signal under Rayleigh fading conditions. The probability density of $x$ is

$$p(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(x - \bar{x})^2}{2\sigma^2} \right]. \quad (2.5-3)$$

The probability that $x$ exceeds the threshold $x_0$ is

$$P_{x_0}(R) = P[x > x_0] = \int_{x_0}^{\infty} p(x) \, dx$$

$$= \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{x_0 - \bar{x}}{\sigma \sqrt{2}} \right). \quad (2.5-4)$$

*The notation dBm used throughout the text stands for dB above one milliwatt.
If we have measured or theoretical values for $\bar{x}$ and for $\sigma$ in the area of interest, we can determine the percent of the area for which the average signal strength exceeds $x_0$. For example, at a radius where the median (and hence the mean of the log-normal) signal strength is $-100$ dBm ($\bar{x} = -100$ dBm at some particular separation distance $R$ and for some radiated power) and assume our system threshold happens to be $-110$ dBm, then, if we assume $\sigma = 10$ dB we have

$$P_{x_0}(R) = \frac{1}{2} + \frac{1}{2} \text{erf}\left(\frac{1}{\sqrt{2}}\right) = 0.84.$$  

2.5.3 Determination of the Coverage Area from a Base Station

It is also of interest to determine the percentage of locations within a circle of radius $R$ in which the received signal strength from a radiating base-station antenna exceeds a particular threshold value. Let us define the fraction of useful service area $F_u$ as that area, within a circle of radius $R$, for which the signal strength received by a mobile antenna exceeds a given threshold $x_0$. If $P_{x_0}$ is the probability that the received signal, $x$, exceeds $x_0$ in an incremental area $dA$, then

$$F_u = \frac{1}{\pi R^2} \int P_{x_0} dA.$$  \hspace{1cm} (2.5-5)

In a real-life situation one would probably be required to break the integration into small areas in which $P_{x_0}$ can be estimated and then sum over all such areas. For purposes of illustration let us assume that the behavior of the mean value of the signal strength follows an $r^{-n}$ law. Thus

$$\bar{x} = \alpha - 10n \log_{10} \frac{r}{R},$$  \hspace{1cm} (2.5-6)

where $\alpha$, expressed in dB, is a constant determined from the transmitter power, antenna heights and gains, and so on. Then

$$P_{x_0} = \frac{1}{2} \left[1 - \frac{1}{2} \text{erf}\left(\frac{x_0 - \alpha + 10n \log_{10} r / R}{\sigma \sqrt{2}}\right)\right].$$  \hspace{1cm} (2.5-7)

Then letting $a = (x_0 - \alpha) / \sigma \sqrt{2}$ and $b = 10n \log_{10} e / \sigma \sqrt{2}$, we get

$$F_u = \frac{1}{2} - \frac{1}{R^2} \int_0^R r \text{erf}\left(a + b \log \frac{r}{R}\right) dr.$$  \hspace{1cm} (2.5-8)
The integral above can be evaluated by substituting \( t = a + b \log(r/R) \), so that

\[
F_u = \frac{1}{2} + \frac{2e^{-2ab}}{b} \int_{-b}^{\infty} e^{-2ut} \text{erf}(t) \, dt. \tag{2.5-9}
\]

From Ref. 55, page 6, No. 1,

\[
F_u = \frac{1}{2} \left[ 1 - \text{erf}(a) + \exp \left( \frac{1 - 2ab}{b^2} \right) \left( 1 - \text{erf} \left( \frac{1 - ab}{b} \right) \right) \right]. \tag{2.5-10}
\]

For example, let us choose \( \alpha = x_0 \) at \( r = R \); then \( a = 0 \) and

\[
F_u = \frac{1}{2} + \frac{1}{2} \exp \left( \frac{1}{b^2} \right) \left[ 1 - \text{erf} \left( \frac{1}{b} \right) \right]. \tag{2.5-11}
\]

Let us further assume that \( n = 3 \) and \( \sigma = 9 \); then \( F_u = 0.71 \), or about 71\% of the area within a circle of radius \( R \) has signal above threshold when half the locations on the circumference have a signal above threshold.

\[\text{Figure 2.5-1} \quad \text{Fraction of total area with signal above threshold, } F_u.\]
For the case where the propagation follows a power law, the important parameter is $\sigma/n$. Figure 2.5-1 is a plot of the fraction of the area within a circle of radius $R$ that has a received signal above a threshold for various fractions of coverage on the circle.

A. P. Barsis has outlined methods for determining coverage when the propagation law is not a simple $r^{-n}$ relationship or when the service area is defined in terms of cochannel interference.

REFERENCES


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