Lecture 9 - Bipolar Junction Transistor Models - Outline

• Announcements
  Handout - Lecture Foils
  First Hour Exam - March 15, 7:30-9:30 pm; thru p-n diodes, PS #4

• BJT operation and optimization: review FAR modeling
  Regions of operation: 1. Forward active; 2. Cut-off; 3. Saturation;
  4. Reverse active
  Designing transistor structures: performance trade-offs, design rules
  Emitter diode model, $\beta_F$ model, $\alpha_F$ model

• Ebers-Moll model for BJTs (using npn as the example)
  Modeling objective: Want $i_E(v_{BE},v_{BC})$ and $i_C(v_{BE},v_{BC})$ given structure
  Approach: Divide and conquer using superposition
  
  $i_E(v_{BE},v_{BC}) = i_E(v_{BE},0) + i_E(0,v_{BC})$
  $i_C(v_{BE},v_{BC}) = i_C(v_{BE},0) + i_C(0,v_{BC})$

  (forward)  (reverse)

  Forward ($v_{BE},0$) and reverse ($0,v_{BC}$) solutions: defects ($\delta$'s), $I_S$'s, $\alpha$'s
  Full model: $I_{ES}, I_{CS}, \alpha_F, \alpha_R$ (forward was also treated in recitations)

• Limitation of FAR models
  Base width modulation, the Early effect
  $\beta$ variation with current: EB SCL recomb., HLI, and IR drops
**npn BJT:** Connecting with the n-channel MOSFET from 6.002

A very similar behavior, and very similar uses.

\[
i_D \approx K [v_{GS} - V_T(v_{BS})]^2/2\alpha
\]

\[
i_B \approx I_{BS} e^{qV_{BE}/kT}
\]

\[
v_{CE} > 0.2 \text{ V}
\]

\[
v_{BE} > 0.6 \text{ V}
\]

\[
v_{DS} > 0.2 \text{ V}
\]
Basic Bipolar Junction Transistor (BJT) - *cross-section*

The heart of the device, and what we will model

An npn BJT
Adapted from Fig. 8.1 in Text
Bipolar Junction Transistors: the carrier fluxes through an npn

Our next task is to determine: Given a structure, what are \( i_E(v_{BE},v_{CE}) \), \( i_C(v_{BE},v_{CE}) \), and \( i_B(v_{BE},v_{CE}) \)?
**npn BJT:** Well designed structure in FAR

\[ N_{DE} \gg N_{AB}, \ w_E \ll L_{hE}, \ w_B << L_{eB} \]

**Excess Carriers:**

\[ (n_i^2/N_{DE})(e^{qV_{BE}/kT} - 1) \]

**Currents:**

\[ i_E \ [= i_E(1 + \delta_E)] \]

\[ i_{hE} \ [= \delta_E i_E] \]

\[ i_C \ [= i_E(1 - \delta_B) \approx i_E] \]

\[ i_B \ [= i_E - (-i_C) \approx i_E \delta_E] \]
**npu BJT, cont.: Resulting F.A.R. model**

With $v_{BE} > 0$, $v_{BC} \leq 0$ (and neglecting $I_{CS}$), we found:

$$i_E = -I_{ES}\left[e^{qV_{BE}/kT} - 1\right]$$
$$i_C = -\alpha_F i_E = \beta_F i_B$$
$$i_B = i_C/\beta_F$$

where the defects are:

$$\delta_E \equiv \frac{i_{hE}}{i_{cE}} = \frac{D_h}{D_e} \cdot \frac{N_{AB}}{N_{DE}} \cdot \frac{w_{B,\text{eff}}}{w_{E,\text{eff}}}$$
and

$$\delta_B \approx \frac{w_{B,\text{eff}}^2}{2D_e \tau_e} = \frac{w_{B,\text{eff}}^2}{2L_e^2} \ll \delta_E$$

The circuit model that fits this behavior is the following:

With:

$$I_{ES} = Aqn_i^2\left(\frac{D_h}{N_{DE}w_{E,\text{eff}}} + \frac{D_e}{N_{AB}w_{B,\text{eff}}}\right)$$

and with $\alpha_F$ and $\beta_F$ as defined above.

Remember that $\alpha_F$ and $\beta_F$ are related:

$$\alpha_F = \frac{\beta_F}{(\beta_F + 1)}, \quad \beta_F = \frac{\alpha_F}{(1 - \alpha_F)}$$

so the model is totally specified by knowing $I_{ES}$ and either $\alpha_F$ or $\beta_F$. 

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Clif Fonstad, 3/09

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**BJT's:** Looking at the characteristics over the whole voltage range

**The Ebers-Moll model**

Our F.A.R. results are a special case of an important general model that describes the BJT in all of its operating regions. It is the "Ebers-Moll" model.

We can readily solve the general flow problem and get a general expression for the BJT characteristics, following the process we used to look at the F.A.R., but in the Ebers-Moll model we make use of superposition and solve the problem with each "excitation" applied separately, and then combine the results:

\[
i_E(v_{BE}, v_{BC}) = i_E(v_{BE}, 0) + i_E(0, v_{BC})
\]

\[
i_C(v_{BE}, v_{BC}) = i_C(v_{BE}, 0) + i_C(0, v_{BC})
\]

Flow problems are linear so we can use superposition, but we do have to be a bit careful because the boundary conditions at the junctions are non-linear functions of \(v_{BE}\) and \(v_{CE}\). The solution is to put a non-zero bias on only one junction at a time.
BJT's: The Ebers-Moll model, cont.

Using superposition, we first find the currents when $v_{BC} = 0$ and $v_{BE}$ is arbitrary, and repeat the process with $v_{BE} = 0$ and $v_{CE}$ arbitrary.

We call the solution for $v_{BC} = 0$ the "**forward**" solution:

$$i_E(v_{BE},0) = -I_{ES}(e^{qv_{BE}/kT} - 1)$$ and $$i_C(v_{BE},0) = -\alpha_F i_E(v_{BE},0)$$

with

$$I_{ES} = Aq n_i^2 \left( \frac{D_h}{N_{DE}w_{E,eff}} + \frac{D_e}{N_{AB}w_{B,eff}} \right), \quad \alpha_F = \frac{1-\delta_B}{1+\delta_E}, \quad \delta_E = \frac{D_h}{D_e} \cdot \frac{N_{AB}}{N_{DE}} \cdot \frac{w_{B,eff}}{w_{E,eff}}, \quad \delta_B \approx \frac{w_{B,eff}^2}{2L_e^2}$$

And the solution for $v_{BE} = 0$ is the "**reverse**" solution:

$$i_C(0,v_{BC}) = -I_{CS}(e^{qv_{BC}/kT} - 1)$$ and $$i_E(0,v_{BC}) = -\alpha_R i_C(0,v_{BC})$$

with

$$I_{CS} = Aq n_i^2 \left( \frac{D_h}{N_{DC}w_{C,eff}} + \frac{D_e}{N_{AB}w_{B,eff}} \right), \quad \alpha_R = \frac{1-\delta_B}{1+\delta_C}, \quad \delta_C = \frac{D_h}{D_e} \cdot \frac{N_{AB}}{N_{DC}} \cdot \frac{w_{B,eff}}{w_{C,eff}}, \quad \delta_B \approx \frac{w_{B,eff}^2}{2L_e^2}$$

Superimposing these results gives us the full Ebers-Moll model:

$$i_E(v_{BE},v_{BC}) = -I_{ES}(e^{qv_{BE}/kT} - 1) + \alpha_R I_{CS}(e^{qv_{BC}/kT} - 1)$$

$$i_C(v_{BE},v_{BC}) = \alpha_F I_{ES}(e^{qv_{BE}/kT} - 1) - I_{CS}(e^{qv_{BC}/kT} - 1)$$

The best way to remember the Ebers-Moll model is as a circuit; this is shown on the next foil.
**BJT's: The Ebers-Moll model, cont.**

Schematically, the forward and backward portions are shown below:

Forward:

\[ I_{ES} = Aq n_i^2 \left( \frac{D_h}{N_{DE} w_{E,\text{eff}}} + \frac{D_e}{N_{AB} w_{B,\text{eff}}} \right) \]

\[ \beta_F = \frac{(1 - \delta_B)}{(\delta_E + \delta_B)} \]

\[ \alpha_F = \frac{(1 - \delta_B)}{(1 + \delta_E)} \]

\[ \delta_E \approx \frac{i_{BE}}{i_{EE}} = \frac{D_h \cdot N_{AB}}{D_e \cdot N_{DE}} \cdot \frac{w_{B,\text{eff}}}{w_{E,\text{eff}}} \]

\[ \delta_B \approx \frac{w_{B,\text{eff}}^2}{2D_e \tau_e} = \frac{w_{B,\text{eff}}^2}{2L_e} \]

Combined they form the full Ebers-Moll model:

Reverse:

\[ I_{CS} = Aq n_i^2 \left( \frac{D_h}{N_{DC} w_{C,\text{eff}}} + \frac{D_e}{N_{AB} w_{B,\text{eff}}} \right) \]

\[ \beta_R = \frac{(1 - \delta_B)}{(\delta_C + \delta_B)} \]

\[ \alpha_R = \frac{(1 - \delta_B)}{(1 + \delta_C)} \]

\[ \delta_C \approx \frac{i_{hC}}{i_{Ce}} = \frac{D_h \cdot N_{AB}}{D_e \cdot N_{DE}} \cdot \frac{w_{B,\text{eff}}}{w_{C,\text{eff}}} \]

\[ \delta_B \approx \frac{w_{B,\text{eff}}^2}{2D_e \tau_e} = \frac{w_{B,\text{eff}}^2}{2L_e} \]

**Note:**

\[ i_F = -i_E(v_{BE},0) \]

and \[ i_R = -i_C(0,v_{BC}). \]
BJT's: The Gummel-Poon model.

Another common model can be obtained from the Ebers-Moll model is the Gummel-Poon model:

Forward:

\[ I_S = \frac{\beta_F}{(\beta_F + 1)} I_{ES} = \frac{\beta_R}{(\beta_R + 1)} I_{CS} \]

\[ = \alpha_F I_{ES} = \alpha_R I_{CS} \]

Reverse:

\[ I_S = \frac{\beta_F}{(\beta_F + 1)} I_{ES} = \frac{\beta_R}{(\beta_R + 1)} I_{CS} \]

\[ = \alpha_F I_{ES} = \alpha_R I_{CS} \]

Combined they form the Gummel-Poon model:

• Aside from the historical interest, another value this has for us in 6.012 is that it is an interesting exercise to show that the two forward circuits above are equivalent.
BJT's: Looking at the characteristics over the whole voltage range

Regions of operation:

The Ebers-Moll and the Gummel-Poon large signal BJT models cover positive and negative biases on both junctions.

They thus cover more than just the forward active region that we care about most.

There is a reverse active region, making a total of four regions of operation.

Bipolar junction transistors can be operated in reverse, i.e. with the base-emitter junction reverse biased and the base-collector junction forward biased, but it is usually not optimized for this mode of operation and typically $\beta_R \ll \beta_F$. 

\[ i_C = \beta_F i_B \]

\[ i_C = -(\beta_R + 1) i_B \]
BJT's: FAR characteristics

Input curve

\[ i_B \approx I_{BS} e^{q V_{BE}/kT} \]

\[ V_{CE} > 0.2 \text{ V} \]

Cutoff

0.6 V

\[ V_{BE} \]

FAR

Output family

\[ i_C \approx \beta_F i_B \]

Saturation

\[ V_{CE} > 0.2 \text{ V} \]

0.2 V

Cutoff

\[ V_{BE} \leq 0.6 \text{ V} \]

Saturation:

\[ V_{CE} > 0.2 \text{ V} \]

Forward active region:

\[ V_{BE} > 0.6 \text{ V} \]

\[ V_{BC} < 0.4 \text{ V} \]

\( i_R \) is negligible

Other regions:

\[ V_{BE} < 0.6 \text{ V} \]

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Lecture 9 - Slide 12
**nnpn BJT:** Equivalent FAR models

\[ I_{ES} = Aq n_i^2 \left( \frac{D_i}{N_{DE} w_{E,eff}} + \frac{D_c}{N_{AB} w_{B,eff}} \right) \]

\[ \alpha_F = \beta_F / (\beta_F + 1) \]

\[ \beta_F = \frac{\alpha_F}{1 - \alpha_F} \]

\[ I_{BS} = I_{ES} / (\beta_F + 1) \]

A useful model using a break-point diode:

\[ i_D \approx \frac{V_{AB}}{I_S} \]

This is a very useful model to use when finding the bias point in a circuit.
nnp BJT Output IV Plot
BJT's, review: Limitations of the large signal model

Limitations of our junction model - their impact on BJT characteristics

- **Base width modulation, the Early effect and Early voltage:**
  The width of the depletion region at the B-C junction increases as $v_{CE}$ increases and the effective base width, $w_{B,eff}$ gets smaller, thereby increasing $\beta$ and, in turn, $i_C$.

- **Punch through:** *base width modulation taken to the limit*
  When the depletion region at the B-C junction extends all through the base all the way to the collector. Punch through has a similar effect on the characteristics as does B-C junction reverse breakdown.

In 6.012 we only take the Early effect into account in our small signal linear equivalent circuit models.
BJT's, review: Limitations of the large signal model

Limitations of our junction model - the impact on BJT characteristics

We looked at this figure when we discussed our diode junction models:

- **Large forward bias:**
  - High level injection (c)
  - Series voltage drop (d)

- **Large reverse bias:**
  - Reverse breakdown

- **Very low bias levels:**
  - SCL generation and recombination (a, e)

Ref: Figure 18 in S. M. Sze, "Physics of Semiconductor Devices" 1st. Ed (Wiley, 1969)

In a BJT these lead to $\beta_F$ decrease at high and at low current levels

Reverse breakdown limits $|V_{CE}|$
BJT's, review: Limitations of the large signal model

Limitations of our junction model - the impact on BJT characteristics

- Beta roll-off at high and low collector currents
- B-C junction breakdown; base punch through

- In 6.012 we assume a constant $\beta$
- We do not include breakdown explicitly in our models.
npn BJT $B_F$ vs $I_C$ Plot
npn BJT Gummel Plot
6.012 - Microelectronic Devices and Circuits

Lecture 9 - Bipolar Junction Transistor Models - Summary

- BJT operation and optimization

  **Optimum design:**
  \[ N_{DE} >> N_{AB} \] to make \( \delta_E \) small
  \[ w_B \] small to make \( \delta_B \) small
  \[ N_{AB} >> N_{DC} \] to make \( w_B^* \) insensitive to \( v_{CE} \)

  npn preferred over pnp because \( D_e > D_h, \mu_e > \mu_h \)

- Devices and models best in F.A.R.

  **Large-signal FAR models:**
  \[ \beta_F = \alpha_F/(1 - \alpha_F) \]
  \[ I_{BS} = (1 - \alpha_F)I_{ES} \]
  \[ = I_{ES}/(\beta_F + 1) \]

- Ebers-Moll Model

  **Defects:**
  - Emitter defect, \( \delta_E = (D_h N_{AB} w_B^*/D_e N_{DE} w_E^*) \) (ideally should be \( \ll 1 \))
  - Base defect, \( \delta_B = (w_B^2/2L_e^2) \) (often negligibly small)
  - Collector defect, \( \delta_C = (D_h N_{AB} w_B^*/D_e N_{DC} w_C^*) \) (might be big)

  **Eqns:**
  \[ i_E(v_{BE}, v_{BC}) = -I_{ES}(e^{v_{BE}/kT} - 1) + \alpha_R I_{CS}(e^{v_{BC}/kT} - 1) \]
  \[ i_C(v_{BE}, v_{BC}) = \alpha_F I_{ES}(e^{v_{BE}/kT} - 1) - I_{CS}(e^{v_{BC}/kT} - 1) \]

  with \( \alpha_F = [(1 - \delta_B)/(1 + \delta_E)] \) \( \approx (1 - \delta_E) \) if \( \delta_B \) is negligible \( \approx 1 \)

  \[ \alpha_R = [(1 - \delta_B)/(1 + \delta_C)] \]

  \[ I_{ES} = A q n_i^2 [(D_h/E N_{DE} w_E^*) + (D_e/B N_{AB} w_B^*)] \]
  \[ I_{CS} = A q n_i^2 [(D_h/E N_{DC} w_C^*) + (D_e/B N_{AB} w_B^*)] \] (Note: \( \alpha_F I_{ES} = \alpha_R I_{CS} \))