6.012 - Electronic Devices and Circuits
Lecture 2 - Uniform Excitation - Outline

• Announcements
  Handouts - Lecture Outline, Review, and Summary

• Review (of Lecture 1 and Recitation 2)
  Carrier concentrations in TE given the doping level
  What happens above and below room temperature?

• Uniform excitation of uniform samples: drift
  Drift motion: carrier velocity verses field
  Mobility, Drift currents, Conductivity
  Impact of temperature on mobility, conductivity
  Integrated circuit resistors

• Uniform excitation: optical generation
  Generation/recombination in TE
  Uniform optical generation - external excitation
  Population excesses, p’ and n’, and their transients
  Low level injection; minority carrier lifetime

• Uniform excitation - applied field and optical generation
  Photoconductivity, photoconductors
Extrinsic Silicon: Given \( N_a \) and \( N_d \), what are \( n_o \) and \( p_o \)?

**Equation 1 - Charge conservation** (the net charge is zero):

\[
q(p_o - n_o + N_d^+ - N_a^-) = 0 \approx q(p_o - n_o + N_d - N_a)
\]

First equation

**Equation 2 - Law of Mass Action** (the np product is constant in thermal equilibrium):

\[
n_o p_o = n_i^2(T)
\]

Second equation

Where does this last equation come from? Consider the reaction:

Electron + Hole \( \leftrightarrow \) Completed bond

The Law of Mass Action in chemistry tells us the concentrations of the reactants and products are related by:

\[
\frac{[\text{Electron}][\text{Hole}]}{[\text{Completed bond}]} = k(T)
\]

Using \([\text{Electron}] = n_o\) and \([\text{Hole}] = p_o\), and recognizing most of the bonds are still completed and \([\text{Completed bond}]\) is essentially a constant, we have

\[
n_o p_o = [\text{Completed bond}] k(T) \approx A k(T) = n_i^2(T)
\]
Extrinsic Silicon, cont: Given \( N_a \) and \( N_d \), what are \( n_o \) and \( p_o \)?

Combine the two equations:

\[
\left( \frac{n_i^2}{n_o} - n_o + N_d - N_a \right) = 0
\]

\[
n_o^2 - (N_d - N_a)n_o - n_i^2 = 0
\]

Solving for \( n_o \) we find:

\[
n_o = \frac{(N_d - N_a) \pm \sqrt{(N_d - N_a)^2 + 4n_i^2}}{2} = \frac{(N_d - N_a)}{2} \left[ 1 \pm \sqrt{1 + \frac{4n_i^2}{(N_d - N_a)^2}} \right]
\]

\[
\approx \frac{(N_d - N_a)}{2} \left[ 1 \pm \left( 1 + \frac{2n_i^2}{(N_d - N_a)^2} \right) \right]
\]

Note: Here we have used \( \sqrt{1 + x} \approx 1 + x/2 \) for \( x \ll 1 \)

This expression simplifies nicely in the two cases we commonly encounter:

Case I - n-type: \( N_d > N_a \) and \( (N_d - N_a) \gg n_i \)

Case II - p-type: \( N_a > N_d \) and \( (N_a - N_d) \gg n_i \)

Fact of life: It is almost impossible to find a situation which is not covered by one of these two cases.
Extrinsic Silicon, cont.: solutions in Cases I and II

**Case I - n-type:** \( N_d > N_a, (N_d - N_a) >> n_i \) "n-type Si"

Define the net donor concentration, \( N_D \):

\[
N_D \equiv (N_d - N_a)
\]

We find:

\[
\begin{align*}
  n_o & \approx N_D, \\
  p_o & = n_i^2(T)/n_o \approx n_i^2(T)/N_D
\end{align*}
\]

In Case I the concentration of electrons is much greater than the concentration of holes. Silicon with net donors is called "n-type". The electrons are the majority carriers; the holes the minority carriers.

**Case II - p-type:** \( N_a > N_d, (N_a - N_d) >> n_i \) "p-type Si"

Define the net acceptor concentration, \( N_A \):

\[
N_A \equiv (N_a - N_d)
\]

We find:

\[
\begin{align*}
  p_o & \approx N_A, \\
  n_o & = n_i^2(T)/p_o \approx n_i^2(T)/N_A
\end{align*}
\]

In Case II the concentration of holes is much greater than the concentration of electrons. Silicon with net acceptors is called "p-type". The holes are the majority carriers; the electrons the minority carriers.
Variation of carrier concentration with temperature

(Note: for convenience we assume an n-type sample)

- **Around R.T.**
  - Full ionization
  - Extrinsic doping
  \[ N_d^+ \approx N_d, \quad N_a^- \approx N_a \]
  \[ \left( N_d^+ - N_a^- \right) \gg n_i \]
  \[ n_o \approx \left( N_d - N_a \right), \quad p_o = \frac{n_i^2}{n_o} \]

- **At very high T**
  - Full ionization
  - Intrinsic behavior
  \[ N_d^+ \approx N_d, \quad N_a^- \approx N_a \]
  \[ n_i \gg \left| N_d^+ - N_a^- \right| \]
  \[ n_o \approx p_o \approx n_i \]

- **At very low T**
  - Incomplete ionization
  - Extrinsic doping, but with carrier freeze-out
  \[ N_d^+ \ll N_d \quad \text{(assuming n-type)} \]
  \[ \left| N_d^+ - N_a^- \right| \gg n_i \]
  \[ n_o \approx \left( N_d^+ - N_a^- \right) \ll \left( N_d - N_a \right), \quad p_o = \frac{n_i^2}{n_o} \]
Uniform material with uniform excitations
(pushing semiconductors out of thermal equilibrium)

A. Uniform Electric Field, $E_x$

Drift motion:
Holes and electrons acquire a constant net velocity, $s_x$, proportional to the electric field:

$$s_{ex} = -\mu_e E_x, \quad s_{hx} = \mu_h E_x$$

At low and moderate $|E|$, the mobility, $\mu$, is constant.
At high $|E|$ the velocity saturates and $\mu$ deceases with increasing $|E|$.
Uniform material with uniform excitations
(pushing semiconductors out of thermal equilibrium)

A. Uniform Electric Field, $E_x$, cont.

Drift motion:
Holes and electrons acquire a constant net velocity, $s_x$, proportional to the electric field:

$$s_{ex} = -\mu_e E_x, \quad s_{hx} = \mu_h E_x$$

At low and moderate $|E|$, the mobility, $\mu$, is constant.
At high $|E|$ the velocity saturates and $\mu$ deceases with increasing $|E|$.

Drift currents:
Net velocities imply net charge flows, which imply currents:

$$J_{ex}^{dr} = -q n_o s_{ex} = q\mu_e n_o E_x \quad J_{hx}^{dr} = q p_o s_{hx} = q\mu_h p_o E_x$$

Note: Even though the semiconductor is no longer in thermal equilibrium the hole and electron populations still have their thermal equilibrium values.
Conductivity, $\sigma_0$:

Ohm's law on a microscale states that the drift current density is linearly proportional to the electric field:

$$J_{x}^{dr} = \sigma_0 E_x$$

The total drift current is the sum of the hole and electron drift currents. Using our early expressions we find:

$$J_{x}^{dr} = J_{ex}^{dr} + J_{hx}^{dr} = q\mu_e n_o E_x + q\mu_h p_o E_x = q(\mu_e n_o + \mu_h p_o)E_x$$

From this we see obtain our expression for the conductivity:

$$\sigma_0 = q(\mu_e n_o + \mu_h p_o) \quad [S/cm]$$

Majority vs. minority carriers:

Drift and conductivity are dominated by the most numerous, or "majority," carriers:

For n-type:

$$n_o >> p_o \Rightarrow \sigma_0 \approx q\mu_e n_o$$

For p-type:

$$p_o >> n_o \Rightarrow \sigma_0 \approx q\mu_h p_o$$
Resistance, $R$, and resistivity, $\rho_o$:

Ohm's law on a macroscopic scale says that the current and voltage are linearly related:

$$v_{ab} = R \cdot i_D$$

The question is, "What is $R$?"

We have:

$$J_x^{dr} = \sigma_o \cdot E_x$$

with $E_x = \frac{v_{AB}}{l}$ and $J_x^{dr} = \frac{i_D}{w \cdot t}$

Combining these we find:

$$\frac{i_D}{w \cdot t} = \sigma_o \cdot \frac{v_{AB}}{l}$$

which yields:

$$v_{AB} = \frac{l}{w \cdot t} \cdot \frac{1}{\sigma_o} \cdot i_D = R \cdot i_D$$

where

$$R \equiv \frac{l}{w \cdot t} \cdot \frac{1}{\sigma_o} = \frac{l}{w \cdot t} \cdot \rho_o$$

Note: Resistivity, $\rho_o$, is defined as the inverse of the conductivity:

$$\rho_o \equiv \frac{1}{\sigma_o} \quad \text{[Ohm - cm]}$$
**Integrated resistors**  Our first device!!

**Diffused resistors:** High sheet resistance semiconductor patterns (pink) with low resistance Al (white) "wires" contacting each end.

**Thin-film resistors:** High sheet resistance tantalum films (green) with low resistance Al (white) "wires" contacting each end.
Velocity saturation

The breakdown of Ohm's law at large electric fields.

Above: Velocity vs. field plot at R.T. for holes and electrons in Si (log-log plot). (Fonstad, Fig. 3.2)

Left: Velocity-field curves for Si, Ge, and GaAs at R.T. (log-log plot). (Neaman, Fig. 5.7)
Variation of mobility with temperature and doping levels

Above: $\mu_e$ vs $T$ in Si at several doping levels

Left: $\mu$ vs doping for Si, Ge, and GaAs at R.T. (Neaman, Fig. 5.3)
Variation of mobility with temperature and doping levels in Si near room temperature

Electrons

Holes

Neaman, Fig. 5.2
Having said all of this,…

…it is good to be aware that the mobilities vary with doping and temperature, but in 6.012 we

1. Will use only one value for the hole mobility in Si, and one for the electron mobility in Si, and will not consider the variation with doping. Typically for bulk silicon we use

\[ \mu_e = 1600 \text{ cm}^2/\text{V-s} \quad \text{and} \quad \mu_h = 600 \text{ cm}^2/\text{V-s} \]

2. Will assume uniform temperature (isothermal) conditions and room temperature operation, and

3. Will only consider velocity saturation when we talk about MOSFET scaling near the end of the term.
Uniform material with uniform excitations
(pushing semiconductors out of thermal equilibrium)

B. Uniform Optical Generation, \( g_L (t) \)

The carrier populations, \( n \) and \( p \):

The light supplies energy to "break" bonds creating excess holes, \( p' \), and electrons, \( n' \). These excess carriers are generated in pairs. Thus:

Electron concentration: \( n_o \Rightarrow n_o + n'(t) \)

Hole concentration: \( p_o \Rightarrow p_o + p'(t) \)

with \( n'(t) = p'(t) \)

Generation, \( G \), and recombination, \( R \):

In general:

\[
\frac{dn}{dt} = \frac{dp}{dt} = G - R
\]

\[
\begin{align*}
G > R & \Rightarrow \frac{dn}{dt} = \frac{dp}{dt} > 0 \\
G < R & \Rightarrow \frac{dn}{dt} = \frac{dp}{dt} < 0
\end{align*}
\]

In thermal equilibrium: \( G = R \)

\[
G = G_o \\
R = R_o = n_o p_o r
\]

\[
G = R \Rightarrow G_o = n_o p_o r = n_i^2 r
\]
B. Uniform Optical Generation, $g_L(t)$, cont.

**With optical generation, $g_L(t)$:**

\[ G = G_o + g_L(t) \]
\[ R = n_p r = (n_o + n')(p_o + p')r \]

**thus**

\[ \frac{dn}{dt} = \frac{dp}{dt} = G - R = G_o + g_L(t) - (n_o + n')(p_o + p')r \]

**The question:** Given $N_d$, $N_a$, and $g_L(t)$, what are $n(t)$ and $p(t)$?

**To answer:** Using

(1) \[ \frac{dn}{dt} = \frac{dp}{dt} = \frac{dn'}{dt} = \frac{dp'}{dt} \]

(2) \[ G_o = R_o = n_o p_o r \]

(3) \[ n' = p' \]

gives one equation in one unknown*:

\[ \frac{dn'}{dt} = g_L(t) - (p_o + n_o + n')n'r \]

(* Remember: $n_o$ and $p_o$ are known given $N_d$, $N_a$)
B. Uniform Optical Generation, $g_L(t)$, cont.

This equation is non-linear, and in general hard to solve:

$$\frac{dn'}{dt} = g_L(t) - (p_o + n_o + n')n'r$$

Special Case - Low Level Injection: assume p-type, $p_o >> n_o$

LLI: $n' << p_o$

When LLI holds our equation becomes linear, and solvable:

$$\frac{dn'}{dt} = g_L(t) - p_o n'r$$

$$= g_L(t) - \frac{n'}{\tau_{\text{min}}} \quad \text{with} \quad \tau_{\text{min}} \equiv 1/p_o r$$

This first order differential equation is very familiar to us. The homogeneous solution is:

$$n'(t) = Ae^{-t/\tau_{\text{min}}}$$
Uniform material with uniform excitations
(pushing semiconductors out of thermal equilibrium)

C. Photoconductivity - drift and optical generation

When the carrier populations change…

\[ g_L(t) \Rightarrow n(t) = n_o + n'(t), \quad p(t) = p_o + n'(t) \]

….the conductivity changes:

\[ \sigma(t) = q[\mu_e n(t) + \mu_h p(t)] \]
\[ = q[\mu_e n_o + \mu_h p_o] + q[\mu_e + \mu_h]n'(t) \]
\[ = \sigma_o + \sigma'(t) \]

This change is used in photoconductive detectors to sense light:

\[ i_D(t) = \sigma(t) \frac{w \cdot d}{l} V_{AB} = [\sigma_o + \sigma'(t)] \frac{w \cdot d}{l} V_{AB} \]
\[ = I_D + i_d(t) \quad \text{with} \quad i_d(t) = \sigma'(t) \frac{w \cdot d}{l} V_{AB} \]

\[ g_L(t) \Rightarrow i_d(t) \]
An antique photoconductor at MIT:

A Stanley Magic Door with a lensed photoconductor-based sensor unit.

Do you know where it is on campus?

To see more and learn the answer check the course Stellar website.
Lecture 2 - Uniform Excitation - Summary

- Review
  Frozen out if too cold; intrinsic-like if too hot

- Uniform excitation of uniform samples: drift
  - n, p unchanged; carriers attain constant velocity, s (viscous flow)
  - $s = \mu E$ at low to moderate fields; s saturates at high fields
  - $J_{e,\text{drift}} = q\mu_e n_o E$ [A/cm²], $J_{h,\text{drift}} = q\mu_h p_o E$ [A/cm²]
  - $J_{\text{drift}} = J_{e,\text{drift}} + J_{h,\text{drift}} = q(\mu_e n_o + \mu_h p_o) E = \sigma_o E$  
    Mobility, $\mu$, decreases as temperature goes up from RT

- Uniform excitation: optical generation
  - In TE, $G_o(T) = R_o(T) = n_o p o r(T)$
  - Uniform illumination adds uniform generation term, $g_L(t)$
  - Populations increase: $n_o \rightarrow n_o + n', p_o \rightarrow p_o + p'$, and $n' = p'$
    - $dn'/dt = dp'/dt = G_o(T) + g_L(t) - R(T) = g_L(t) - [np - n_o p_o] r(T)$
    - Focus is on minority $\approx g_L(t) - n'/\tau_{\text{min}}$ if LLI holds, with $\tau_{\text{min}} = [p_o r(T)]^{-1}$

- Uniform excitation: both optical and electrical
  - Photoconductivity: $\sigma_o \rightarrow \sigma_o + \sigma' = q[\mu_e (n_o + n') + \mu_h (p_o + p')]$
    - $\sigma_o + q(\mu_e + \mu_h) p'$
  - Photoconductors: an important class of light detectors