Energy and Noise Revisited

- Constellation diagrams and SNR
- Bit error rate versus SNR
- Shannon Capacity Limit

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Review of Digital Modulation

- Transmitter sends discrete-valued signals over an analog communication channel
- Receiver samples recovered baseband signal
  - Noise and ISI corrupt received signal
- Key techniques
  - Properly design transmit and receive filters for low ISI
  - Sample and slice received signals to detect symbols
A Closer Look at the Transmitter

- Amplitude of I/Q transmit signals impact power of transmitted output
  - Output power is limited due to FCC regulations within a given spectral band
  - Low output power is desirable for portable applications to achieve long battery life
A Constellation View of Transmitter

- Provides intuitive view of relationship between symbol separation and transmitted power
A Constellation View of Receiver

- Provides an intuitive view of relationship between symbol separation, received signal power, and noise
Impact of SNR on Receiver Constellation

- **SNR influenced by transmitted power, distance between transmitter and receiver, and noise**
Impact of Increased Signal on Constellation

- Increase in received signal power leads to increased separation between symbols
  - SNR is improved if noise level unchanged
Quantifying the Impact of Noise

- **Minimum separation between symbols:** $d_{\text{min}}$
- **PDF of noise:** zero mean Gaussian PDF
  - Variance of noise sets the spread of the PDF
- **Bit errors:** occur when noise moves a symbol by a distance more than $d_{\text{min}}/2$
Impact of Reduced SNR

- Lower SNR leads to a reduced value for $d_{\text{min}}$
- Leads to a higher bit error rate
  - Assumes noise variance is unchanged
Impact of Symbol Reduction

- Reducing the number of symbols leads to an increased value for $d_{\text{min}}$
- Leads to a lower bit error rate
  - Assuming SNR remains constant
Can We Estimate Bit Error Rate?

- Bit Error Rate depends on:
  - SNR
    - Received signal power versus noise variance
  - Number of constellation points
    - Sets $d_{\text{min}}$ at a given level of received signal power
Let's Start with a Detailed System View

- Assumptions: No ISI, 4-point constellation
A Closer Examination of Signal and Noise

Communication Channel

Baseband Input

\[ I_{IN} \rightarrow i(t) \rightarrow i_r(t) \rightarrow i(t) \rightarrow I_{OUT} \]

\[ Q_{IN} \rightarrow q(t) \rightarrow q_r(t) \rightarrow q(t) \rightarrow Q_{OUT} \]

Transmit & Receive Pair

\[ i_r(t) \rightarrow d_{min} \rightarrow i_r(t) \]

\[ q_r(t) \rightarrow d_{min} \rightarrow q_r(t) \]

Receiver Output

\[ i_r(t) \rightarrow Q_{received} \rightarrow Q_{OUT} \]

\[ q_r(t) \rightarrow Q_{received} \rightarrow Q_{OUT} \]

Sample & Slice

\[ i_r(t) \rightarrow \text{Slicer} \rightarrow i_{OUT} \]

\[ q_r(t) \rightarrow \text{Slicer} \rightarrow Q_{OUT} \]

Communication Channel for Q Channel

\[ Q_{IN} \rightarrow d_{min}/2 \rightarrow Q_{received} \rightarrow Q_{OUT} \]

Decision Boundary

\[ Q_{signal} = d_{min}/2 \]

PDF of Noise

\[ f_X(x) = \sigma^2 \]

Variance
The Binary Symmetric Channel Model

- Provides a binary signaling model of channel
Computation of SNR

Communication Channel for Q Channel

[Diagram of Q channel with Q_{IN} and Q_{OUT} and decision boundary]

PDF of Received Q Sample

Transmitted 0

f_{Q0}(y)

Transmitted 1

f_{Q1}(x)

- \frac{d_{min}}{2}

0

\frac{d_{min}}{2}

Bit error

Signal Variance

= \left( \frac{d_{min}}{2} \right)^2

Noise Variance

= \sigma^2

\Rightarrow \quad SNR(dB) = 10 \log \left( \frac{(d_{min}/2)^2}{\sigma^2} \right)
Resulting Bit Error Rate Versus SNR

Communication Channel for Q Channel

PDF of Received Q Sample

Transmitted 0

Transmitted 1

Bit Error Rate versus SNR for Q Channel

Note:

- Bit Error Rate = $P_e$
- $SNR \ (dB) = \frac{10 \log \left( \frac{(d_{min}/2)^2}{\sigma^2} \right)}{10}$
- Gaussian PDF for noise
• In 1948, Claude Shannon proved that
  - Digital communication can achieve arbitrary low bit-error-rates if appropriate *coding* methods are employed
  - The capacity of a *Gaussian channel* with bandwidth $BW$ to support arbitrary low bit-error-rate communication is:

  $$C = BW \log_2(1 + SNR) \text{ bits/second}$$
Impact of Channel Bandwidth on Capacity

- An increase in bandwidth by a factor of 2 allows twice the number of bits to be sent in time T
  - Capacity (bits/second) increases linearly with bandwidth

\[ C = BW \log_2(1 + SNR) \text{ bits/second} \]
Impact of SNR on Capacity

\[ C = BW \log_2(1 + SNR) \text{ bits/second} \]

- A high SNR allows more bits to be sent per symbol
  - Adding \( n \) bits requires adding \( 2^n \) constellation points
  - Adding \( n \) bits therefore leads to \( d_{\text{min}} \) being reduced by a factor of \( 2^n \)
  - Capacity increases logarithmically with SNR
**Constellation Design (Symbol Packing)**

- **Objective:** design constellation to maximize $d_{\text{min}}$ while packing as many points in as possible
  - Maximizing $d_{\text{min}}$ achieves lowest *uncoded* bit error rate
  - Maximizing number of constellation points achieves highest *uncoded* data rate (bits/second)
Summary

• Constellation diagrams allow intuitive approach of quantifying *uncoded* bit error rate of a channel
  - Function of SNR and number of constellation points

• A digital communication channel can be viewed in terms of a binary signaling model
  - Focuses attention on key issue of bit error rate

• Coding theoretically allows arbitrary low bit-error-rate performance of a practical digital communication link
  - We will dive more into this topic in the coming weeks....

• Next lecture: summary of Part I (Volt-by-volt)