I/Q Modulation and RC Filtering

- Issues with coherent modulation
- Analog I/Q modulation principles
- RC networks as continuous-time filters
- Differentiation property of Fourier Transform
**AM Modulation and Demodulation**

- **Multiplication (i.e., mixing) operation shifts in frequency**
  - Also creates undesired high frequency components at receiver
- **Lowpass filtering passes only the desired baseband signal at receiver**

What can go wrong here?
Impact of 90 Degree Phase Shift

- If receiver cosine wave turns into a sine wave, we suddenly receive no baseband signal!
  - We apparently need to synchronize the phase of the transmitter and receiver *local oscillators*
  - This is called *coherent* demodulation

- Some key questions:
  - How do we analyze this issue?
  - What would be the impact of a small *frequency offset*?
Frequency Domain Analysis

- When transmitter and receiver local oscillators are matched in phase:
  - Demodulated signal *constructively* adds at baseband
**Impact of 90 Degree Phase Shift**

- When transmitter and receiver local oscillators are 90 degree offset in phase:
  - Demodulated signal *destructively* adds at baseband

**What would happen with a small frequency offset?**
I/Q Modulation

- Consider modulating with both a cosine and sine wave and then adding the results
  - This is known as I/Q modulation
- The I/Q signals occupy the same frequency band, but one is real and one is imaginary
  - We will see that we can recover both of these signals

\[ y(t) = i(t) + j q(t) \]

\[ y(t) = 2\cos(2\pi f_0 t) + 2\sin(2\pi f_0 t) \]

\[ y(t) = Y_i(j2\pi f) + j Y_q(j2\pi f) \]

\[ Y_i(j2\pi f) = I(t)(j2\pi f) \]

\[ Y_q(j2\pi f) = Q(t)(j2\pi f) \]

I stands for in-phase component

Q stands for quadrature component
I/Q Demodulation

- Demodulate with both a cosine and sine wave
  - Both I and Q channels are recovered!
- I/Q modulation allows twice the amount of information to be sent compared to basic AM modulation with same bandwidth

What can go wrong here?
Impact of 90 Degree Phase Shift

- **I and Q channels get swapped at receiver**
  - Key observation: no *information* is lost!

- **Questions**
  - What would happen with a *small* frequency offset?
  - What would happen with a *large* frequency offset?
Summary of Analog I/Q Modulation

- **Frequency domain view**

Baseband Input

- $I_i(j2\pi f)$
- $Q_i(j2\pi f)$

Receiver Output

- $I_r(j2\pi f)$
- $Q_r(j2\pi f)$

- $2\cos(2\pi f_0 t)$
- $2\sin(2\pi f_0 t)$

- $H(j2\pi f)$ Lowpass

- $2\cos(2\pi f_0 t)$
- $2\sin(2\pi f_0 t)$

- $H(j2\pi f)$ Lowpass

- $i(t)$
- $q(t)$

- $i_r(t)$
- $q_r(t)$

- $I(t)$
- $Q(t)$

- $2$ (Red)
- $0$ (Purple)

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$

- $I(t)$
- $Q(t)$

- $I_r(t)$
- $Q_r(t)$

- $2$

- $0$

- $f$

- $t$
• Analyze by first deriving a differential equation relating output and input voltages

\[ I_R(t) = \frac{V_{in}(t) - V_{out}(t)}{R} = C \frac{dV_{out}(t)}{dt} \]

• The filter frequency response is defined as

\[ H(j2\pi f) = \frac{V_{out}(j2\pi f)}{V_{in}(j2\pi f)} \]

• The output voltage corresponds to a scaled and phase shifted version of the input cosine wave

\[ V_{out}(t) = |H(j2\pi f_o)| \cos(2\pi f_o t + \angle H(j2\pi f_o)) \]
Differentiation Property of FT

Fourier Transform Definition

\[ x(t) \iff X(j2\pi f) \]

\[ x(t) = \int_{-\infty}^{\infty} X(j2\pi f) e^{j2\pi ft} df \]

\[ X(j2\pi f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \]

- Derive impact of differentiation

\[ \frac{d}{dt} x(t) = \frac{d}{dt} \int_{-\infty}^{\infty} X(j2\pi f) e^{j2\pi ft} df \]

\[ = \int_{-\infty}^{\infty} j2\pi f X(j2\pi f) e^{j2\pi ft} df \]

\[ \frac{d}{dt} x(t) \iff j2\pi f X(j2\pi f) \]
Derivation of RC Filter Response

![RC Filter Circuit Diagram]

- Apply FT to above differential equation

\[
\frac{V_{in}(j2\pi f) - V_{out}(j2\pi f)}{R} = Cj2\pi f V_{out}(j2\pi f)
\]

\[
\Rightarrow \frac{V_{in}(j2\pi f)}{R} = \left(Cj2\pi f + \frac{1}{R}\right)V_{out}(j2\pi f)
\]

- Filter frequency response is then calculated as

\[
H(j2\pi f) = \frac{V_{out}(j2\pi f)}{V_{in}(j2\pi f)} = \frac{1}{1 + RCj2\pi f}
\]
Magnitude of RC Filter Response

- Define cutoff frequency of filter
  \[ f_c = \frac{1}{2\pi RC} \Rightarrow H(j2\pi f) = \frac{1}{1 + jf/f_c} \]

- Magnitude of response:
  \[ |H(j2\pi f)| = \frac{1}{\sqrt{1 + (f/f_c)^2}} \]
Summary

- Coherent modulation requires synchronized local oscillators at transmitter and receiver
  - Impact of phase offset is to change baseband amplitude
  - Impact of frequency offset is fading (small offset) or catastrophic corruption (large offset) of baseband signal

- I/Q modulation allows twice the amount of information to be sent compared to basic AM
  - Impact of phase offset is to swap I/Q
  - Impact of frequency offset is I/Q swapping (small offset) or catastrophic corruption (large offset) of received signal

- RC networks provide continuous-time filtering

- Upcoming lectures
  - Examine another non-ideality: noise
  - Lay groundwork for digital modulation and the concept of information