Lecture 1 - Compound Semiconductors; Energy bands and carriers I - Outline

- Announcements
  Handouts - General Information; Syllabus; Lecture 1 Notes

- Why we need to look beyond Silicon
  (Why isn't Si enough?)
  Representative applications silicon is not suitable for
  (...at least not yet)

- Which materials are semiconductors?
  (What are our choices?)
  Elemental semiconductors
  Compound semiconductors - binaries: 1. III-V's; 2. II-IV's; 3. IV-VI's; 4. I-VII's
  Alloy semiconductors: 1. Ternaries; 2. Quarternaries; 3. Others: a) More than 4; b) Si-Ge

- Electron energy levels, bands, and states in crystals
  Energy bands
  Density of states
  Electron state occupancy, and $E_F(T)$

- Band-edge plots through homojunction devices
  (Lec. 2 probably)
  Thermal equilibrium: 1. Uniform sample, 2. p-n junction, 3. MOS cap., 4. M-S contact
  Non-equilibrium situations: 1. Optical excitation, 2. Biased p-n junction
  Quasi-Fermi levels: 1. Definition, 2. Significance
## Important properties of silicon

### Physical, structural
- **Crystal structure**: diamond
- **Lattice period (Å)**: 5.431

### Energy levels
- **Energy gap (eV)**: 1.1
- **Band symmetry**: indirect gap
- **Density of states (cm⁻³)**:
  - \(N_c = 2.8 \times 10^{19}\)
  - \(N_v = 1.02 \times 10^{19}\)

### Electrical, charge carriers
- **Electrons**
  - **Low field mobility (cm²/V-s)**: 1450
  - **Critical E-field (V/cm)**: \(10^4\)
  - **Saturation velocity (cm/s)**: \(10^7\)
  - **Effective mass (relative)**:
    - \(m_l = 0.98\)
    - \(m_{lh} = 0.16\)
    - \(m_t = 0.19\)
    - \(m_{hh} = 0.5\)
- **Holes**
  - **Low field mobility (cm²/V-s)**: 450
  - **Critical E-field (V/cm)**: \(5 \times 10^4\)
  - **Saturation velocity (cm/s)**: \(10^7\)
  - **Effective mass (relative)**:
    - \(m_{lh} = 0.16\)
    - \(m_{hh} = 0.5\)

### Optical
- **Absorption edge (\(\lambda_{gap}\))**: 1.1 µm
- **Radiative lifetime (s)**: few ms
- **Typical radiative Efficiency (%)**: <<1%
Things that cannot yet be made from silicon

- **Light emitters**
  - Light emitting diodes, Laser diodes
  - any wavelength

- **Mid- and far-infrared detectors** ($\lambda \geq 1.1 \, \mu m$)
  - Fiber communication wavelengths
    - $\lambda = 1.3$ and $1.55 \, \mu m$
  - Atmospheric windows
    - $\lambda = 3$ to $5 \, \mu m$ and $8$ to $12 \, \mu m$
  - Infrared imaging arrays
  - Thermophotovoltaic cells
    - night vision
    - responding to 500 K black bodies

- **Ultraviolet detectors** ($\lambda \leq 0.5 \, \mu m$)
  - Solar blind detectors
    - no response in visible

- **Optical modulators**
  - Amplitude modulation of light
    - for fiber telecomm

- **Very-high speed electronics**
  - Systems operating at 40 GHz and above
    - for fiber telecomm

- **High temperature electronics**
  - Operable at temperatures above 200°C
    - process monitoring

- **Cryogenic electronics**
  - Operating at 4.2 K and below
    - space instrumentation
**Materials other than Si that are semiconductors:**

- **Binary compounds**

  The choices are many-
  - Column III with column V (the three-fives, III-V's): $A_{III}B_V$
  - Column II with column VI (the two-sixes, II-VI's): $A_{II}B_{VI}$
  - Column IV with Column VI (the four-sixes, IV-VI's): $A_{IV}B_{VI}$
  - Column I with Column VII: $A_IB_{VII}$ (these are insulators)

To help us make sense of all these options we will find that there are clear trends (a method to the madness)

The best way to start is by looking at plots of lattice period vs. energy gap...

<table>
<thead>
<tr>
<th></th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
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<tr>
<td>B</td>
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<td>N</td>
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<td>O</td>
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<td>Ga</td>
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<tr>
<td>Ge</td>
<td>32</td>
<td></td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>As</td>
<td>33</td>
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<td>34</td>
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<tr>
<td>Sb</td>
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<tr>
<td>Se</td>
<td></td>
<td></td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>Cd</td>
<td>48</td>
<td>In</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>In</td>
<td>49</td>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Sn</td>
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<tr>
<td>Hg</td>
<td>80</td>
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<td>81</td>
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<tr>
<td>Tl</td>
<td>81</td>
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<tr>
<td>Pb</td>
<td>82</td>
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<td>83</td>
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<tr>
<td>Bi</td>
<td>83</td>
<td></td>
<td>84</td>
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Elemental Semiconductors: Column IV:

Diamond lattice

<table>
<thead>
<tr>
<th>Element</th>
<th>Energy gap $E_g$ [eV]</th>
<th>Lattice period $a$ [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>5.5</td>
<td>0.357</td>
</tr>
<tr>
<td>Si</td>
<td>1.12</td>
<td>0.543</td>
</tr>
<tr>
<td>Ge</td>
<td>0.67</td>
<td>0.564</td>
</tr>
<tr>
<td>Sn</td>
<td>0.08</td>
<td>0.649</td>
</tr>
</tbody>
</table>

Energy gap vs. lattice period plot

Note: C and Sn take several crystal forms. The diamond form of each is semiconducting.
Compound Semiconductors: Before there was Si-Ge there were the III-V’s, II-VI’s, and IV-VI’s

{Note: Strictly speaking, Si-Ge, is an alloy, but we’ll call it a compound semiconductor like everyone else.}

Diamond lattice

Many III-V’s and II-VI’s crystallize in the zinc blende lattice (GaAs shown)

Other lattices we’ll see are the wurtzite and rock salt.

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Lecture 1 - Slide 6
Binary Compound Semiconductors: Zinc-blende III-V's II-VI's

- **ZnSe**
- **AIP**
- **GaP**
- **AlAs**
- **CdS**
- **ZnTe**
- **GaAs**
- **InP**
- **AlSb**
- **CdSe**
- **CdTe**
- **GaSb**
- **InAs**
- **InSb**
- **Si**
- **Ge**

**Axes:**
- **X-axis:** Lattice period, $a$ (nm)
- **Y-axis:** Energy gap, $E_g$ (eV)
- **Z-axis:** Energy wavelength, $\lambda_g$ (μm)
### Binary Compound Semiconductors: Zinc-blende III-V's II-VI's

<table>
<thead>
<tr>
<th>Material System</th>
<th>Semiconductor Name</th>
<th>Symbol</th>
<th>Crystal Lattice Structure</th>
<th>Period(A)</th>
<th>Energy Band Gap(eV)</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>III-V</td>
<td>Aluminum phosphide</td>
<td>AlP</td>
<td>Z</td>
<td>5.4510</td>
<td>2.43</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Aluminum arsenide</td>
<td>AlAs</td>
<td>Z</td>
<td>5.6605</td>
<td>2.17</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Aluminum antimonide</td>
<td>AlSb</td>
<td>Z</td>
<td>6.1355</td>
<td>1.58</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Gallium phosphide</td>
<td>GaP</td>
<td>Z</td>
<td>5.4512</td>
<td>2.26</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Gallium arsenide</td>
<td>GaAs</td>
<td>Z</td>
<td>5.6533</td>
<td>1.42</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Gallium antimonide</td>
<td>GaSb</td>
<td>Z</td>
<td>6.0959</td>
<td>0.72</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Indium phosphide</td>
<td>InP</td>
<td>Z</td>
<td>5.8686</td>
<td>1.35</td>
<td>d</td>
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<tr>
<td></td>
<td>Indium arsenide</td>
<td>InAs</td>
<td>Z</td>
<td>6.0584</td>
<td>0.36</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Indium antimonide</td>
<td>InSb</td>
<td>Z</td>
<td>6.4794</td>
<td>0.17</td>
<td>d</td>
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<tr>
<td>II-VI</td>
<td>Zinc sulfide</td>
<td>ZnS</td>
<td>Z</td>
<td>5.420</td>
<td>3.68</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Zinc selenide</td>
<td>ZnSe</td>
<td>Z</td>
<td>5.668</td>
<td>2.71</td>
<td>d</td>
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<tr>
<td></td>
<td>Zinc telluride</td>
<td>ZnTe</td>
<td>Z</td>
<td>6.103</td>
<td>2.26</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Cadmium sulfide</td>
<td>CdS</td>
<td>Z</td>
<td>5.8320</td>
<td>2.42</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Cadmium selenide</td>
<td>CdSe</td>
<td>Z</td>
<td>6.050</td>
<td>1.70</td>
<td>d</td>
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<tr>
<td></td>
<td>Cadmium telluride</td>
<td>CdTe</td>
<td>Z</td>
<td>6.482</td>
<td>1.56</td>
<td>d</td>
</tr>
</tbody>
</table>

**Key:** Z = zinc blende; i = indirect gap, d = direct gap
### Additional Semiconductors: Wurzite III-V's and II-VI's

#### Lead Salts (IV-VI's), Column IV

<table>
<thead>
<tr>
<th>Material System</th>
<th>Semiconductor Name</th>
<th>Symbol</th>
<th>Crystal Lattice Structure</th>
<th>Period(A)</th>
<th>Energy Band Gap(eV)</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>III-V (nitrides)</td>
<td>Aluminum Nitride</td>
<td>AlN</td>
<td>W</td>
<td>a = , c =</td>
<td>6.2</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Gallium Nitride</td>
<td>GaN</td>
<td>W</td>
<td>a = 3.189, c = 5.185</td>
<td>3.36</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Indium Nitride</td>
<td>InN</td>
<td>W</td>
<td>a = , c =</td>
<td>0.7</td>
<td>d</td>
</tr>
<tr>
<td>II-VI (wurtzite)</td>
<td>Zinc Sulfide</td>
<td>ZnS</td>
<td>W</td>
<td>a = 3.82, c = 6.28</td>
<td>3.68</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Cadmium Sulfide</td>
<td>CdS</td>
<td>W</td>
<td>a = 4.16, c = 6.756</td>
<td>2.42</td>
<td>d</td>
</tr>
<tr>
<td>IV-VI</td>
<td>Lead Sulfide</td>
<td>PbS</td>
<td>R</td>
<td>5.9362</td>
<td>0.41</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Lead Selenide</td>
<td>PbSe</td>
<td>R</td>
<td>6.128</td>
<td>0.27</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>Lead Telluride</td>
<td>PbTe</td>
<td>R</td>
<td>6.4620</td>
<td>0.31</td>
<td>d</td>
</tr>
<tr>
<td>IV</td>
<td>Diamond</td>
<td>C</td>
<td>D</td>
<td>3.56683</td>
<td>5.47</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Silicon</td>
<td>Si</td>
<td>D</td>
<td>5.43095</td>
<td>1.124</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Germanium</td>
<td>Ge</td>
<td>D</td>
<td>5.64613</td>
<td>0.66</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Grey Tin</td>
<td>Sn</td>
<td>D</td>
<td>6.48920</td>
<td>0.08</td>
<td>d</td>
</tr>
<tr>
<td>IV-IV</td>
<td>Silicon Carbide</td>
<td>SiC</td>
<td>W</td>
<td>a = 3.086, c = 15.117</td>
<td>2.996</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Silicon-Germanium</td>
<td>Si_x Ge_{1-x}</td>
<td>Z</td>
<td>vary with x (i.e. an alloy)</td>
<td>i</td>
<td></td>
</tr>
</tbody>
</table>

**Key:** Z = zinc blende, W = wurtzite, R = rock salt; i = indirect gap, d = direct gap

C. G. Fonstad, 2/07
Binary Compound Semiconductors: mobility trends

Sze HSSD Fig 1

Sze HSSD Fig 2
Materials other than Si that are semiconductors:

- **Binary compounds**
  - Most have direct bandgaps. (**very important to optoelectronic device uses**)
  - They cover a wide range of bandgaps, but only at discrete points.
  - They follow definite trends
  - They can be grown in bulk form and cut into wafers.
  - We still need more….

- **Ternary alloys**
  - Not compounds themselves, but alloys of two binary compounds with one common element. (**ternary compounds are of limited interest**)
  - Ternary alloys have two elements from one column, one from another and there are two options: (**III-V examples**)
    
    $A_{\text{III}(1-x)}B_{\text{III}(x)}C_V = [A_{\text{III}}C_V]_{(1-x)} + [B_{\text{III}}C_V]_{(x)}$
    
    $A_{\text{III}}B_{\text{V}(1-y)}C_{V(y)} = [A_{\text{III}}B_V]_{(1-y)} + [A_{\text{III}}C_V]_{(y)}$
  
  With ternary alloys we have access to a continuous range of bandgaps
Ternary Alloy Semiconductors: 3 III-V examples, AlGaAs, InGaAs, InAlAs

Three III-V ternaries: InGaAs, AlGaAs, InAlAs

<table>
<thead>
<tr>
<th>Lattice period (Angstroms)</th>
<th>InGaAs</th>
<th>AlGaAs</th>
<th>InAlAs</th>
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<tbody>
<tr>
<td>5.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.9</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>6.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.1</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Energy Gap (eV)

- InGaAs
- AlGaAs
- InAlAs

C. G. Fonstad, 2/07
Ternary trends:

Most properties, such as effective mass, vary quadratically and monotonically with alloy fraction.

Alloy scattering is largest near a 50% mix and → transport properties tend to not vary monotonically.
Materials other than Si that are semiconductors:

- **Ternary alloys**
  
  Give us access to continuous ranges of bandgaps, but as $E_g$ varies so in general does $a$.

  Substrates are always binary and only come at discrete $a$'s.

  Thus to grow heterostructures we need different $E_g$ layers, all with the same $a$, and ternaries don't do the full job.

  (Note: AlGaAs is an important exception; since it is intrinsically lattice-matched to GaAs it was used in the first heterostructure work. However, soon more was needed...)

- **Quarternary alloys**
  
  Quarternaries mix 4 elements - there are 2 types:

  **(III-V examples)**

  1. 2 elements from one column, 2 from the other:
     
     \[
     A_{\text{III}}(1-x) B_{\text{III}}(x) C_{\text{V}}(1-y) D_{\text{V}}(y) = [A_{\text{III}} C_{\text{V}}]_{(1-x)(1-y)} + [A_{\text{III}} D_{\text{V}}]_{(1-x)y} + [B_{\text{III}} C_{\text{V}}]_{x(1-y)} + [B_{\text{III}} D_{\text{V}}]_{xy}
     \]

  2. 3 elements from one column, 1 from the other:
     
     \[
     A_{\text{III}}(1-x-y) B_{\text{III}}(x) C_{\text{III}}(y) D_{\text{V}} = [A_{\text{III}} D_{\text{V}}]_{(1-x-y)} + [B_{\text{III}} D_{\text{V}}]_{(x)} + [C_{\text{III}} D_{\text{V}}]_{(y)}
     \]

     \[
     A_{\text{III}} B_{\text{V}}(1-x-y) C_{\text{V}}(x) D_{\text{V}}(y) = [A_{\text{III}} B_{\text{V}}]_{(1-x-y)} + [A_{\text{III}} C_{\text{V}}]_{(x)} + [A_{\text{III}} D_{\text{V}}]_{(y)}
     \]

  With quarternary alloys we have access to ranges of in materials that are all lattice-matched to a binary substrate.
III-V quarternary alloys: InGaAlAs
III-V quarternary alloys: more examples InGaAsP and AlGaAsSb
III-V quarternary alloys: another example  GaInAsSb
III-V quarternary alloys: still more GaAlAsP and GaAlInP
The III-V wurtzite quarternary: GaInAlN

Sze PSD Fig 2a

Energy gap, $E_g$ (eV)

Lattice period, $a$ (nm)

Energy wavelength, $\lambda_g$ ($\mu$m)
So...where are we?

Are all these semiconductors important?

- All have uses, but some are more widely used than others
  - GaAs-based heterostructures
  - InP-based heterostructures
  - Misc. II-IVs, III-Vs, and others

- Important Binaries
  - **GaAs**
  - **InP**
  - **GaP**

- Important Ternaries and Quaternaries
  - **AlGaAs on GaAs**
  - **GaAsP on GaAs**
  - **HgCdTe on CdTe**
  - **InGaAsP, InGaAlAs on InP**
  - **InGaAlAs on InP**
  - **InGaAs on GaAs, InP**
  - **InGaAsP on GaAs**
  - **GaInAlN on various substrts.**

- Important Binaries
  - **substrates, MESFETs**
  - **substrates**
  - **red, green LEDs**

- Important Ternaries and Quaternaries
  - **HBTs, FETs, optoelectronic (OE) devices**
  - **red, amber LEDs**
  - **IR imagers**
  - **OEs for fiber telecomm.**
  - **ditto**
  - **ohmic contacts, quantum wells**
  - **red and IR lasers, detectors**
  - **green, blue, UV LEDs, lasers**
Note: The conduction and valence bands are in general not symmetrical, i.e., $m_{cd} \neq m_{vd}$. 

\[ \rho(E > E_c) \equiv \rho_c(E) = \frac{\sqrt{2(m_{cd}^*)^3 (E - E_c)}}{\pi^2 \hbar^3} \]

\[ \rho(E_v > E) \equiv \rho_v(E) = \frac{\sqrt{2(m_{vd}^*)^3 (E_v - E)}}{\pi^2 \hbar^3} \]
**Fermi Function and Fermi Level - summary of key points**

**Fermi function:** In thermal equilibrium, the probability of an energy level at $E$ being occupied is given by the Fermi function, $f(E)$:

$$f(E) = \frac{1}{1 + e^{(E-E_f)/kT}}$$

where $E_f$ is the Fermi energy, or “level.” In thermal equilibrium $E_f$ is constant and not a function of position.

The Fermi function has the following useful properties:

- $f(E) \approx e^{-(E-E_f)/kT}$ for $(E - E_f) \gg kT$
- $f(E) \approx 1 - e^{(E-E_f)/kT}$ for $(E - E_f) \ll -kT$
- $f(E_f) = \frac{1}{2}$ for $E = E_f$

These relationships tell us that the population of electrons decreases exponentially with energy at energies much more than $kT$ above the Fermi level, and similarly that the population of holes (empty electron states) decreases exponentially with energy when more than $kT$ below the Fermi level.
Visualizing the Fermi Function - variation with $E$ and $T$

At $E_f$

$$f(E_f) = \frac{1}{2}$$

Well below $E_f$

$$1 - f(E) \approx e^{\frac{E-E_f}{kT}}$$

(= probability of finding a hole)

Well above $E_f$

$$f(E) \approx e^{-\frac{E-E_f}{kT}}$$

(= probability of finding an electron)

When $T$ approaches $0$ K:

$$\lim_{T \to 0} f(E) = 1 \quad \text{for} \quad E < E_f$$

$$f(E) = \frac{1}{2} \quad \text{at} \quad E = E_f$$

$$\lim_{T \to 0} f(E) = 0 \quad \text{for} \quad E > E_f$$

C. G. Fonstad, 2/07
Filling states - combining the density of states and Fermi functions

\[ n(E) = \rho(E) \cdot f(E) \]
Filling states, cont. - \( p_o, \, n_o, \, \text{and} \, E_f \) given \( N_d \) and \( N_a \)

Once \( E_f \) is known, \( p_o \) and \( n_o \) can be calculated, and the starting point for calculating \( E_f \) is charge neutrality:

\[
p_o - n_o + N_d^+ - N_a^- = 0
\]

Each factor in this expression can be written in terms of \( f(E) \):

\[
p_o = \int_{-\infty}^{E_v} \rho_v(E) \left[ 1 - f(E) \right] dE \quad N_d^+ = N_d \left[ 1 - f_d(E_d) \right]
\]

\[
n_o = \int_{E_c}^{\infty} \rho_c(E) f(E) dE \quad N_a^- = N_a \cdot f_a(E_a)
\]

Inserting these gives one equation in one unknown, \( E_f \):

\[
\int_{-\infty}^{E_v} \rho_v(E) \left[ 1 - f(E) \right] dE - \int_{E_c}^{\infty} \rho_c(E) f(E) dE + N_d \left[ 1 - f_d(E_d) \right] - N_a \cdot f_a(E_a) = 0
\]

Solving this equation and actually calculating \( E_f \) is tedious in general, and not terribly enlightening, but we can describe \( E_f \) qualitatively many times, and we can find an analytical expression for \( E_f \) in very important special cases. We’ll look at the latter first…
Variation of Fermi level with temperature

(Note: for convenience we assume an n-type sample, no acceptors)

- Around R.T.
  Tail of $f(E)$ extends above $E_c$
  Full ionization
  Extrinsic doping

- At very low $T$
  $f(E)$ almost a step function
  Incomplete ionization
  Freeze-out of electrons

Fermi energy, $E_f$

$E_i < E_f < E_d$

$E_d < E_f < E_c$

- At very high $T$
  $f(E)$ has long tails extending above $E_c$ and below $E_v$
  Band-to-band thermal excitation of electrons
  Intrinsic behavior
Variation of carrier concentration with temperature
(Note: for convenience we assume an n-type sample)

- **Around R.T.**
  - Full ionization
  - Extrinsic doping
  \[
  N_d^+ \approx N_d, \quad N_a^- \approx N_a
  \]
  \[
  \left( N_d^+ - N_a^- \right) \gg n_i
  \]
  \[
  n_o \approx \left( N_d - N_a \right), \quad p_o = n_i^2 / n_o
  \]

- **At very high T**
  - Full ionization
  - Intrinsic behavior
  \[
  N_d^+ \approx N_d, \quad N_a^- \approx N_a
  \]
  \[
  n_i \gg \left| N_d^+ - N_a^- \right|
  \]
  \[
  n_o \approx p_o \approx n_i
  \]

- **At very low T**
  - Incomplete ionization
  - Extrinsic doping, but with carrier freeze-out
  \[
  N_d^+ \ll N_d \quad \text{(assuming n - type)}
  \]
  \[
  \left| N_d^+ - N_a^- \right| \gg n_i
  \]
  \[
  n_o \approx \left( N_d^+ - N_a^- \right) \ll \left( N_d - N_a \right), \quad p_o = n_i^2 / n_o
  \]
Variation of Fermi level with temperature
(This time for a variety of doping conditions)

- **Samples**
  - A. Intrinsic doping
  - B. N-type, donors only
  - C. P-type, acceptors only
  - D. N-type, compensated
Effective densities of states:

When the Fermi level is many $kT$ away from either band edge [i.e., $(E_c - E_f) \gg kT$ and $(E_f - E_v) \gg kT$], then we can approximate and simplify the two integrals in the expressions for $n_o$ and $p_o$. Looking first at $n_o$:

$$n_o = \int_{E_c}^{\infty} \rho_c(E) f(E) \, dE = \int_{E_c}^{\infty} \frac{\rho_c(E)}{1 + e^{(E_f - E)/kT}} \, dE$$

$$\approx \int_{E_c}^{\infty} \rho_c(E) e^{-(E_f - E)/kT} \, dE = \left[ \int_{E_c}^{\infty} \rho_c(E) e^{-(E_f - E)/kT} \, dE \right] e^{-(E_c - E_f)/kT}$$

We now define the effective density of states for the conduction band, $N_c$, as

$$N_c \equiv \int_{E_c}^{\infty} \rho_c(E) e^{-(E_c - E)/kT} \, dE$$

and write $n_o$ simply as:

$$n_o \approx N_c e^{-(E_c - E_f)/kT}$$

Using the same approach to define an effective density of states for the valence band, $N_v$, we have also:

$$p_o \approx N_v e^{-(E_f - E_v)/kT} \quad \text{with} \quad N_v \equiv \int_{-\infty}^{E_v} \rho_v(E) e^{(E_f - E_v)/kT} \, dE$$
With these approximate expressions for $n_o$ and $p_o$ (which are valid only when the Fermi level is several $kT$ away from the band edges) the equation for $E_f$,

$$p_o - n_o + N_d^+ - N_a^- = 0$$

becomes:

$$N_v e^{-(E_f - E_v)/kT} - N_c e^{-(E_c - E_f)/kT} + N_d [1 - f_d(E_d)] - N_a f_a(E_a) = 0$$

It is still not simple to find $E_f$, but it is much easier.

If the energy bands are parabolic, i.e., when the density of states depends quadratically on the energy away from the band edge, we find simple relationships between the densities of states and the effective masses:

If $\rho(E) = \sqrt{2(m^*_{ed})^3 (E - E_c)}/\pi^2 \hbar^3$ then $N_c = 2 \left[ 2\pi m^*_{ed} kT/\hbar^2 \right]^{3/2}$

and if $\rho(E) = \sqrt{2(m^*_{hd})^3 (E_v - E)}/\pi^2 \hbar^3$ then $N_v = 2 \left[ 2\pi m^*_{hd} kT/\hbar^2 \right]^{3/2}$