Lecture 15 - Quantum effects in heterostructures, II - Outline

- Continue wells, wires, and boxes from L 14

- Coupled wells and superlattices
  
  Two coupled quantum wells:
  1. Energy level system
  2. Impact of separation/coupling

  Superlattices:
  1. Energy levels
  2. Applications

Specialized multiple well structures and their applications
  1. Quantum Cascade lasers (Intro only; more Recitation 11 - Prof. Hu)
  2. QWIP detectors (Intro only; more Recitation 12)
  3. RTDs (Topic of Recitation 7)
Electrons - quantum mechanical description:

Quantum mechanically, an electron with mass, \( m_e \), is described by a wave function, \( \Psi(x,y,z,t) \), which is in turn related to the probability of finding the electron at the position \((x,y,z)\) at time \(t\). The wave function satisfies the Schrödinger Equation:

\[
\frac{\hbar^2}{2m_e} \Delta^2 \Psi - U(x,y,z) \Psi = \frac{\hbar}{i} \frac{\partial \Psi}{\partial t}
\]

where \( U(x,y,z) \) is the potential energy of the system.

This equation has solutions of the form

\[
\Psi(x,y,z,t) = \psi(x,y,z) e^{-iEt/\hbar}
\]

where \( \psi(x,y,z) \) satisfies

\[
\nabla^2 \psi + \frac{2m_e}{\hbar^2} \left[ E - U(x,y,z) \right] \psi = 0
\]

In one dimension\(^*\) this equation is

\[
\frac{d^2 \psi}{dx^2} + \frac{2m_e}{\hbar^2} \left[ E - U(x) \right] \psi = 0
\]

\(^*\) We will restrict ourselves to one-dimension in the following derivations.
Electron moving in free space - quantum mechanical view:

In free space, $U(x) = 0$ and we can have:

$$\frac{d^2\psi}{dx^2} + \frac{2m_eE}{\hbar^2}\psi = \frac{d^2\psi}{dx^2} + k^2\psi = 0$$

where we have defined $k$ through the equality $k^2 = 2m_eE/\hbar^2$. The solution of this equation is of the form:

$$\psi(x) = A_+ e^{ikx} + A_- e^{-ikx}$$

And thus the entire wave function is of the form:

$$\Psi(x, t) = A_+ e^{ikx - iEt/\hbar} + A_- e^{-ikx - iEt/\hbar}$$

$$= A_+ e^{-i(\omega t - kx)} + A_- e^{-i(\omega t + kx)}$$

From this we see that the wave function is two plane waves traveling in opposite directions with angular frequency, $\omega$, and wave numbers, $\pm k$. This is consistent with the uncertainty principle of quantum mechanics which says that if we know the energy of a particle exactly we can’t say anything about when it had that energy. To know where an electron is when, we need to form a packet of plane waves spanning a range of energies in such a way as to yield a finite probability of finding the electron localized in space.
Quantum heterostructures - coupled quantum wells

**Two isolated quantum wells:** identical, isolated levels

Wavefunctions (identical)

**Two coupled quantum wells:** isolated levels split into two levels for the combined system, slightly shifted from the original position

Symmetric Wavefunction

Assymetric Wavefunction
Quantum heterostructures - superlattices

Isolated quantum well:

\[ n = 1 \]

N coupled quantum wells: isolated levels split into N levels for the combined system, all slightly shifted from the original position and forming a mini-band of states.
Superlattices and mini-bands

\[ \Delta E_{c} = 500 \text{ meV} \]

[Y.H. Wang, S.S. Li and Pin Ho, "Voltage-tunable dual-mode operation InAlAs/InGaAs quantum well infrared photodetector for narrow- and broadband detection at 10 \( \mu \text{m} \)," Appl. Phys. Lett. 62 (1993) 621]
Quantum heterostructures - applications

Applications (mainly of quantum wells):
- Laser diode active layers (Lectures 19-21)
- QWIP structure (Recitation 12)
- Quantum cascade laser structure (Recitation 11)
- Resonant tunneling diodes (tomorrow)

Quantum wires and boxes - comment

We can make nearly ideal quantum wells with extreme dimension control and reproducibility.
When it comes to wires and boxes we lack the dimensional control needed to produce them controllably. Thus it is not possible (yet) to make many wonderful devices one might imagine making from perfect wires and boxes. At the same time, there are still things one can do with the quantum wires and boxes we can make, so all is not lost....stay tuned.
Quantum cascade lasers -
using inter-subband transitions

Right: The basic unit of the quantum cascade laser heterostructure.

Above: Schematic illustration of stage-to-stage cascade

Right: Detailed QW structure of a cascade period
**Quantum Well Infrared Photodectors - QWIPs**

Above: Schematic illustration of QWIP structure and function.

Right: Energy separation between \( n = 1 \) and \( 2 \) levels in quantum wells with indicated aluminum fractions and well widths.
Resonant tunneling diodes

Conduction band edge profiles:

**Unbiased:**

**Biased:**

A. At resonance
   100% transmission

B. Above resonance
   no transmission

**I-V characteristics:**
Quantum Tunneling through Single Barriers

Transmission probabilities - Ref: Jaspirit Singh, Semiconductor Devices - an introduction, Chap. 1

Rectangular barrier

\[ T = \frac{4}{\left\{4 \cosh^2 \alpha d + [(\alpha / k) - (k / \alpha)] 2 \sinh^2 \alpha d \right\}} \]

where \( k^2 = 2m^* (E - E_c) / \hbar^2 \) and \( \alpha^2 = 2m_o [\Delta E_c - (E - E_c)] / \hbar^2 \)
Common 1-d potential energy landscapes, cont.

A one-dimensional resonant tunneling barrier:

Classically, electrons with \(0 < E < \Delta E_c\) can again not pass from one side to the other, while those with \(E > \Delta E_c\) do not see the barriers at all.

Quantum mechanically, electrons with \(0 < E < \Delta E_c\) with energies that equal energy levels of the quantum well can pass through the structure unattenuated; while a fraction of those with \(E > \Delta E_c\) will be reflected by the steps.