Lecture 20 - Laser Diodes 1 - Outline

- **Stimulated emission and optical gain**
  Absorption, spontaneous emission, stimulated emission
  Threshold for optical gain

- **Laser diode basics**
  Lasing and conditions at threshold
  Threshold current density
  Differential quantum efficiency

- **In-plane laser cavity design** (as far as we get; we'll finish in Lect. 21)
  Vertical structure: homojunction
  double heterojunction
  quantum well, wire, dot; quantum cascade
  Lateral definition: stripe contact
  buried heterostructure
  shallow rib
  End-mirror design: cleaved facet
  etched facet
  distributed feedback, Bragg reflector
Laser diodes: comparing LEDs and laser diodes

Light emitting diodes vs. Laser diodes

**LEDs** are based on *spontaneous* emission, and have
1. A broad output beam that is hard to capture and focus
2. A relatively broad spectral profile
3. Low to moderate overall efficiency
4. Moderate to high speed ($\approx 1/\tau_{\text{min}}$)

**Laser Diodes** are based on *stimulated* emission, and have the opposite characteristics
1. Narrow, highly directed output
2. Sharp, narrow emission spectrum
3. High differential and overall efficiency
4. High to very high speed

**Stimulated emission** occurs when a passing photon triggers the recombination of an electron and hole, with emission of a second photon with the same frequency (energy), momentum, and phase.
Laser diodes: achieving stimulated gain

To understand what is necessary to obtain net optical gain, rather than net absorption, we consider optical transitions between two levels in a solid ($E_1$ and $E_2$), and we look at three transitions occurring with the absorption or emission of photons:

1. from $E_1$ to $E_2$ due to absorption
2. from $E_2$ to $E_1$ due to spontaneous emission
3. from $E_2$ to $E_1$ due to stimulated emission

We model the rate of each process using the Einstein A and B coefficients, and then find when the probability is higher that a photon passing will stimulate emission than be absorbed.
Laser diodes: achieving stimulated gain, cont.

In a semiconductor we consider one state, $E_1$, to be in the valence band, and the other, $E_2$ to be in the conduction band:

The rates these processes occur depend on the populations:

Absorption rate, $R_{ab}$: photon pop. $\times$ $E_1$ pop. $\times$ $E_2$ empty state pop.

Spontaneous emission rate, $R_{sp}$: $E_2$ pop. $\times$ $E_1$ empty state pop.

Stimulate emission rate, $R_{st}$: $E_2$ pop. $\times$ $E_1$ empty state pop. $\times$ photon pop.
Laser diodes: achieving stimulated gain, cont

Absorption rate:

\[ R_{ab} = B_{12} \cdot f_1 \cdot \rho_v(E_1) \cdot (1 - f_2) \cdot \rho_c(E_2) \cdot \rho_p(E_2 - E_1) \]

where

- \( B_{12} \): transition probability for absorption
- \( \rho_v(E_1) \): valence band density of states at \( E_1 \)
- \( \rho_c(E_2) \): conduction band density of states at \( E_2 \)
- \( \rho_p(E_2 - E_1) \): density of photons with correct energy
- \( f_1 \): Fermi function evaluated at \( E_i \)

\[ f_i = 1 / \left( e^{E_i - E_{fi}} + 1 \right) \]

where

- \( E_{fi} \): quasi-Fermi level for level \( i \)

Spontaneous emission rate:

\[ R_{sp} = A_{21} \cdot f_2 \cdot \rho_c(E_2) \cdot (1 - f_1) \cdot \rho_v(E_1) \]
**Laser diodes:** achieving stimulated gain, cont

In the last equation we introduced:

- $A_{21}$: transition probability for spontaneous emission

**Stimulated emission rate:**

$$R_{st} = B_{21} \cdot f_2 \cdot \rho_c(E_2) \cdot (1 - f_1) \cdot \rho_v(E_1) \cdot \rho_p(E_2 - E_1)$$

where

- $B_{21}$: transition probability for stimulated emission

Note, finally, that in these expressions the Fermi function is evaluated either in the conduction band ($i = 2$) or valence band ($i = 1$):

$$f_1 = 1 / \left( e^{E_1 - E_{fv}} + 1 \right), \quad f_2 = 1 / \left( e^{E_2 - E_{fc}} + 1 \right)$$

The coefficients, $A_{21}$, $B_{12}$, and $B_{21}$, are related, as we can see by looking at thermal equilibrium, where

$$R_{ab} = R_{sp} + R_{st}, \quad E_{fv} = E_{fc}, \quad \rho_p(E_i) = \frac{8 \pi r_o^3}{h^3 c^3} E_i^2 \frac{1}{\left( e^{E_i / kT} - 1 \right)}$$

**Note:** $\rho_p(E_i) =$ photons/cm$^3$-eV

C. G. Fonstad, 4/07

Lecture 20 - Slide 6
Laser diodes: achieving stimulated gain, cont

Proceeding in this we we find:

\[ B_{12} = B_{21}, \quad \text{and} \quad A_{21} = \frac{8\pi r_o^3 E_i^2}{h^3 c^3} B_{21} \]

***************

Now we are ready to find the condition for optical gain, which we take as when the probability of stimulated emission is greater than that for absorption. Looking back at our equations, we find \( R_{st} > R_{ab} \) leads to:

\[ B_{21} \cdot f_2 \rho_c \cdot (1 - f_1) \rho_v \cdot \rho_p (E_2 - E_1) > B_{12} \cdot f_1 \rho_v \cdot (1 - f_2) \rho_c \cdot \rho_p (E_2 - E_1) \]

Canceling equivalent terms yields:

\[ f_2 (1 - f_1) > f_1 (1 - f_2) \]

and substituting the appropriate Fermi functions gives us:

\[ E_{fc} - E_{fv} > (E_2 - E_1) = \hbar \nu \geq E_g \]
Laser diodes: achieving stimulated gain, cont.

Our conclusion is that we will have net optical gain, i.e., more stimulated emission than absorption, when we have the quasi-Fermi levels separated by more than the band gap. This in turn requires high doping and current levels. It is the equivalent of population inversion in a semiconductor:

\[ E_{fc} - E_{fv} > E_g \]

**************

Next we relate the absorption coefficient, \( \alpha \), to \( R_{ab} \), \( R_{st} \), and \( R_{sp} \). A bit of thought shows us that we can say:

\[
R_{ab}(E) > \left[ R_{st}(E) + R_{sp}(E) \right] \approx R_{st}(E) \rightarrow \alpha(E) > 0 \quad \text{Net loss}
\]

\[
R_{ab}(E) < \left[ R_{st}(E) + R_{sp}(E) \right] \approx R_{st}(E) \rightarrow \alpha(E) < 0 \quad \text{Net gain}
\]

\[
R_{ab}(E) = \left[ R_{st}(E) + R_{sp}(E) \right] \approx R_{st}(E) \rightarrow \alpha(E) = 0 \rightarrow E = E_{fc} - E_{fv}
\]

Notes: Spontaneous emission is negligible because it is randomly directed. It starts the lasing process, but it does not sustain it. The point at which \( \alpha = 0 \) is called the transparency point.
**Laser diodes:** optical gain coefficient, $g(E)$

The negative of the absorption coefficient is defined as the gain coefficient:

$$g(E) \equiv -\alpha(E)$$

Writing the light intensity in terms of $g(E)$ we have:

$$L(E, x) = L_o(E)e^{-\alpha(E)z} = L_o(E)e^{g(E)z}$$

Stimulated recombination is proportional to the carrier populations, and in a semiconductor one carrier is usually in the minority and its population is the one that changes significantly with increasing current injection. If we assume p-type material, we have:

$$g > 0 \rightarrow n > n_{tr}$$

To first order, the gain will be proportional to this population, to the extent that it exceeds the transparency level:

$$g \approx G(n - n_{tr})$$
**Laser diodes: threshold current**

We not look at a laser diode and calculating the threshold current for lasing, and the light-current relationship.

Consider the following cavity:

![Diagram of a laser cavity](image)

- **Reflection at facet,** $R_1$
- **Additional loss,** $\alpha_{\text{loss}}$
- **R_2**

**Photons inside, reflecting between the end faces, stimulate more photons (i.e., emission).**

**Excess carriers are injected into this volume somehow, typically by the current across a p-n junction, and at high current there will be gain.**

Lasing will be sustained when the optical gain exceeds the optical losses for a round-trip in the cavity.

The **threshold current** is the current level above which this occurs.
Laser diodes: threshold current, cont.

Track the light intensity on a full circuit, beginning with $I_o$ just inside the facet at $z = 0^+$, and directed to the right:

1. At $z = 0^+$, directed to the right, $I(0^+) = I_o$
2. At $z = L^-$, directed to the right, $I(L^-) = I_o e^{(g-\alpha_l)L}$
3. At $z = L^-$, directed to the left, $I(L^-) = R_2 I_o e^{(g-\alpha_l)L}$
4. At $z = 0^+$, directed to the left, $I(0^+) = R_2 I_o e^{(g-\alpha_l)2L}$
5. At $z = 0^+$, directed to the right, $I(0^+) = R_1 R_2 I_o e^{(g-\alpha_l)2L}$

For sustained lasing we must have the intensity after a full circuit (5) be equal to, or greater than, the initial intensity (1):

$$R_1 R_2 I_o e^{(g-\alpha_l)2L} \geq I_o$$
Laser diodes: threshold current, cont.

This leads us to identify the threshold gain, \( g_{th} \):

\[
g_{th} = \alpha_l + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)
\]

To relate this threshold gain to current we recall that the gain is proportional to the carrier population in excess of the transparency value,

\[
g \approx G(n - n_{tr}) = G' \Gamma(n - n_{tr})
\]

where \( G' \): the portion of \( G \) due to material parameters alone

\( \Gamma \): the portion of \( G \) due to geometrical factors (i.e., the overlap of the optical mode and the active medium)

and that the population will in general be proportional to the current:

\[
n \approx K_i_D
\]

where \( K \): a proportionality factor that depends on the device structure, which we will determine in specific situations later
Laser diodes: threshold current, cont.

Writing $g$ in terms of $i_D$, and setting it equal to $g_{th}$, yields:

$$g_{th} = G' \Gamma (Ki_D - n_{tr}) = \alpha_l + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$$

The diode current that corresponds to this threshold gain is defined to be the threshold current, $I_{th}$:

$$i_D \equiv i_D(g_{th}) = I_{th} = \frac{1}{K} \left( \frac{1}{G' \Gamma} \left[ \alpha_l + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \right] + n_{tr} \right)$$

(This will take on more meaning as we look at specific laser diode geometries and quantify the various parameters.)

A final useful observation is that the mirror reflectivity term in these equations can be viewed as a mirror loss coefficient, $\alpha_m$:

$$\alpha_m \equiv \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$$
Laser diodes: threshold current, cont.

Above threshold, essentially all of the additional excitation fuels stimulated recombination, and \( n' \) stays fixed at its threshold value. So too does \( E_{fn} - E_{fp} \), which implies that the junction voltage is also pinned.

The current-voltage and power-current characteristics of a laser diode thus have the following forms:
**Laser diodes: output power, \( P_{\text{opt}} \), and external differential quantum efficiency, \( \eta_{ed} \)**

To calculate the optical output power, \( P_{\text{opt}} \), we begin with several points:

**First**, we recall that a particle flux density can be written in terms of a particle density times their velocity. This holds for photons as well, and the velocity is the mode, or "group" velocity:

\[
F_{ph}(x, y) = v_g \ N_{ph}(x, y)
\]

**Second**, we recall that the rate of change of a photon population with time at a given point, is the photon flux density times the absorption coefficient, \( \alpha \):

\[
\frac{\partial N_{ph}(x, y)}{\partial t} = -\alpha \ v_g \ N_{ph}(x, y)
\]

**Finally**, we note that the output power will be the total flux of photons emitted times the energy per photon, \( h \nu \).

\[
P_{out} = h \nu \int F_{ph}(x, y) \, dx \, dy = h \nu \ F_{ph, \text{out}, \text{tot}}
\]
Laser diodes: $P_{\text{opt}}$ and $\eta_{\text{ed}}$, cont.

We next move inside the laser diode and look at the photon population there. The total number of photons inside the laser will be

$$N_{\text{ph, in, tot}} = \int N_{\text{ph, in}}(x, y) \, dx \, dy \, dz = L \int N_{\text{ph, in}}(x, y) \, dx \, dy$$

This photon population is decreasing because photons are being absorbed internally and emitted from the ends of the cavity at a rate given by the photon flux times the effective absorption coefficient:

$$\text{Loss} : \quad \int (\alpha_l + \alpha_m) v_g N_{\text{ph, in}}(x, y) \, dx \, dy \, dz = (\alpha_l + \alpha_m) v_g N_{\text{ph, in, tot}}$$

Note that the loss out the end mirrors, $\alpha_m$, is the output we are trying to calculate!

and it is increasing because the diode current exceeds the threshold current, and the gain exceeds the threshold gain:

$$\text{Photon generation} : \quad \int g v_g N_{\text{ph, in}}(x, y) \, dx \, dy \, dz = \frac{(i_D - I_{\text{th}})}{q} \eta_i$$
Laser diodes: \( P_{\text{opt}} \) and \( \eta_{\text{ed}}, \) cont.

In the last equation, \( \eta_i \), is the current utilization efficiency, the fraction feeding stimulated recombination.

In the steady state the loss equals the generation:

\[
(\alpha_l + \alpha_m) v_g N_{\text{ph,in,tot}} = \frac{(i_D - I_{\text{th}})}{q} \eta_i
\]

Thus the total photon population inside the laser diode is:

\[
N_{\text{ph,in,tot}} = \frac{(i_D - I_{\text{th}})}{q(\alpha_l + \alpha_m) v_g} \eta_i
\]

The total photon flux emitted from the laser diode output mirrors is the portion of the "loss" due to \( \alpha_m \):

\[
P_{\text{opt}} = h \nu \alpha_m v_g N_{\text{ph,in,tot}} = h \nu \frac{(i_D - I_{\text{th}})}{q} \frac{\alpha_m}{(\alpha_l + \alpha_m)} \eta_i
\]
**Laser diodes:** $P_{opt}$ and $\eta_{ed}$, cont.

We next introduce the extraction efficiency, $\eta_e$:

$$\eta_e \equiv \frac{\alpha_m}{(\alpha_l + \alpha_m)}$$

With this we write the output power as:

$$P_{opt} = h\nu \left( \frac{i_D - I_{th}}{q} \right) \eta_e \eta_i$$

**Note:** This is the total output power from both ends of the laser.

**$P_{opt}$: output per facet**

To see how much comes out the individual faces we can return to the figure on foil 16, and now look at the emitted intensities, i.e. beams 6 and 7 in the figure below:
Laser diodes: $P_{\text{opt}}$: output per facet

Earlier we had

1. At $z = 0^+$, directed to the right, $I(0^+) = I_o$
2. At $z = L^-$, directed to the right, $I(L^-) = I_o e^{(g-\alpha_1) L}$
3. At $z = L^-$, directed to the left, $I(L^-) = R_2 I_o e^{(g-\alpha_1) L}$
4. At $z = 0^+$, directed to the left, $I(0^+) = R_2 I_o e^{(g-\alpha_1) 2L}$
5. At $z = 0^+$, directed to the right, $I(0^+) = R_1 R_2 I_o e^{(g-\alpha_1) 2L}$

And we concluded that above threshold we have: $R_1 R_2 e^{(g-\alpha_1) 2L} = 1$

Now we focus our attention on the emitted light, 6 and 7:

6. At $z = 0^-$, emission to the left, $I(0^-) = (1 - R_1) R_2 I_o e^{(g-\alpha_1) 2L}$
7. At $z = L^+$, emission to the right, $I(L^+) = (1 - R_2) I_o e^{(g-\alpha_1) L}$
Laser diodes: \( P_{\text{opt}}: \) output per facet, cont.

We use these last expressions, along with \( R_1 R_2 e^{(g-\alpha_i)2L} = 1 \), to find the fraction of the emission emitted to the left, the fraction to the right, and the ratio of the two:

\[
\text{Fraction emitted to left: } \frac{I(0^-)}{I(0^-) + I(L^+)} = \left[\frac{(1 - R_1)}{\sqrt{R_1}}\right] + \left[\frac{(1 - R_2)}{\sqrt{R_2}}\right]
\]

\[
\text{Fraction emitted to right: } \frac{I(L^+)}{I(0^-) + I(L^+)} = \left[\frac{(1 - R_2)}{\sqrt{R_2}}\right] + \left[\frac{(1 - R_1)}{\sqrt{R_1}}\right]
\]

\[
\text{Ratio of left to right: } \frac{I(0^-)}{I(L^+)} = \left[\frac{(1 - R_1)}{\sqrt{R_1}}\right] = \frac{(1 - R_1)}{(1 - R_2)} \cdot \frac{\sqrt{R_2}}{\sqrt{R_1}}
\]

We can make the following observations:

If the two end-faces have equal reflectivities, then half the power will come out each end.

If one end is much more highly reflecting than the other, then little power will come out it, and all of the power will come out the lower reflectivity facet.

If the reflectivities of the ends differ, but not by a large amount, then the division of output is bit more complicated.
Laser diodes: \( P_{\text{opt}} \) and \( \eta_{\text{ed}} \), cont.

Next we turn to the external differential quantum efficiency.

The external differential quantum efficiency is defined as the ratio between the number of photons emitted per unit time, divided by the number of carriers crossing the diode junction per unit time:

\[
\eta_{\text{ed}} = \frac{\Delta(\text{# of photons out/unit time})}{\Delta(\text{# of carriers across junction/unit time})}
\]

In terms of the output power and diode current this is:

\[
\eta_{\text{ed}} = \frac{\Delta(P_{\text{opt}} / h\nu)}{\Delta(i_D/q)} = \frac{q}{h\nu} \frac{dP_{\text{opt}}}{di_D}
\]

Using the result on an earlier foil, we find:

\[
\eta_{\text{ed}} = \eta_e \eta_i
\]
Laser diodes: device design and optimization

With this general background, we now turn to looking at specific device design and the evolution of diode laser structures over time.

- We will begin looking at evolution of active region design and the vertical device structure.

- Next we turn to lateral definition of the cavity.

- Finally we look at defining the ends of the cavity.

- This initial discussion (Lec. 21) will focus on edge-emitting in-plane lasers. After this we will turn to vertically emitting and vertical cavity devices (in Lec. 22).

- At the very end we will discuss laser diode modulation.
**Laser diodes: vertical design evolution**

**Double heterostructure:**
The first major advance in laser diode design was the double heterostructure geometry which confines the carriers and the light to the same region.

The threshold current density is approximately

\[ J_{th} = \frac{q n_{crit} d}{\tau_{min}} \]

As predicted, and shown to the right, \( J_{th} \) decreases linearly with \( d \) until the guide layer is too thin to confine the light, at which point the overlap decreases and the threshold increases.

Ref: Yariv, Optical Electronics, Fig. 15-13(a).
Laser diodes: vertical design evolution

Double heterostructure: carriers and light are confined by the same narrow bandgap layer. The threshold decreases with d until the optical mode spills out.

Separate confinement DH: the waveguide and carrier confinement functions are done by different layers. The overlap is less, but the threshold is still reduced.

Quantum well: the overlap is less than in SCDH, but there is a net win because the quantum well transitions are stronger.
Laser diodes: Separate confinement issues

Overlap estimate: because the optical mode is peaked in the center of the waveguide the overlap of the mode and the inverted carrier population is greater than might first be expected. In the situation illustrated below, the inner layer is 1/3 the thickness, but the overlap integral is only reduced by 1/2.

Waveguide portion options: the optical confinement/waveguide layer is often graded by some means so the carriers can fall into the active or QW layers more easily:

- Simple SCDH structure
- Linearly grading
- Parabolic grading
- Step grading
Photoluminescence - light emission from silicon - the latest!!

Several years ago when we discussed light emission from semiconductors the question was asked, "Modern silicon wafer boules are extremely pure and have very low defect levels. Shouldn't this silicon show efficient emission?"

We concluded that contrary to popular lore, the photoluminescent efficiency in the bulk of this silicon should be high, but that perhaps non-radiative surface recombination kept the overall efficiency low.

The next Monday experimental data was published* showing this is true:

**Very efficient light emission from bulk crystalline silicon**

Thorsten Trupke, Jianhua Zhao, Aihua Wang, Richard Corkish, and Martin A. Green
Centre for Third Generation Photovoltaics, University of New South Wales, Sydney, NSW, 2052, Australia

(Received 20 January 2003; accepted 10 March 2003)

Due to its indirect bandstructure, bulk crystalline silicon is generally regarded as a poor light emitter. In contrast to this common perception, we report here on surprisingly large external photoluminescence quantum efficiencies of textured bulk crystalline silicon wafers of up to 10.2% at $T = 130$ K and of 6.1% at room temperature. Using a theoretical model to calculate the escape probability for internally generated photons, we can conclude from these experimental figures that the radiative recombination probability or internal luminescence quantum efficiency exceeds 20% at room temperature. © 2003 American Institute of Physics. [DOI: 10.1063/1.1572473]

They conclude: "...silicon can be a very efficient light emitter if the surface recombination is reduced by efficient surface passivation..." and "...the EQE of a large number of commercially available float-zone n- and p-type silicon wafers with different resistivities was found to be on the order of a few percent. This shows that our findings are generally valid for highest-quality silicon."


Just 4 years ago!!
Photoluminescence - light emission from silicon - cont.

The figures from the article:

**Fig. 1**: PL external quantum efficiency (EQE) for a 500 µm thick sample at 130 and 297 K. Note the saturation at high excitation levels.

**Fig. 2**: PL intensity vs. T for four pump levels (15.8, 29.3, 72.9, and 117 mW/cm²). Note the drop at low temperature when phonon population decrease dominates.

**Fig. 3**: Modulated PL intensity vs. the pump modulation frequency at several temperatures. Note that the response is slow, but also that the speed of a device can be much faster because in a device carriers can be injected and extracted actively, and the minority carrier lifetime need not be the limiting factor.


C. G. Fonstad, 4/07