• **Introduction**
  - **Structure** - What are we talking about?
  - **Behaviors:** Ohmic, rectifying, neither

• **Band picture in thermal equilibrium** (Establishing the baseline)
  - **Ideal junction** - no surface states
  - **Real junctions** - surface states and Fermi level pinning

• **Applying voltage bias (i-v and c-v)** (Where it gets interesting, i.e. useful)
  - Forward bias, current flow
    1. General comments; 2. Thermionic emission theory;
    3. Drift-diffusion theory; 4. Real junctions
  - Reverse bias, image-force lowering
  - **Switching dynamics**
    1. Step response; 2. High frequency response

• **Applications** (Benefiting from these simple structures)
  - **Ohmic contacts**
  - **Doping profiling**
  - **Shunt diodes**
  - **FET gate (MESFETs)**
  - **UV photodiodes**
Unbiased p-n junction

Electrostatic potential:

Band-edge plot:

Vacuum ref. now -q\(\phi(x)\)

Valence band close to \(E_f\) so \(p_o\) is large

Band edges away from \(E_f\) so region is depleted

Conduction band close to \(E_f\) so \(n_o\) is large
**p-n Junctions** - progression of the barrier with decreasing separation
Band-edge profiles - key observations

The electrostatics relationships from 6.012 are still useful and provide the basic foundation for profiling:

Electrostatics:

\[
\rho(x) = q \left[ p_o(x) - n_o(x) + N_D^+(x) - N_A^-(x) \right]
\]

\[
\phi(x) = -\int F(x)dx = -\int \int \frac{\rho(x)}{\varepsilon}dxdx
\]

The band edges follow the profile of the electrostatic potential energy, \(-q\phi(x)\):

Band edges:

\[
E_c(x) = -q\phi(x) - \chi(x)
\]

\[
E_c(x) = -q\phi(x) - \chi(x) - E_g(x)
\]

Not functions of x in homogeneous devices

The Fermi level, \(E_f\):

1. Constant in thermal equilibrium, TE
2. Near CB \(\Rightarrow n \text{ large}\)
3. Near VB \(\Rightarrow p \text{ large}\)
The Fermi energy - showing it is constant in TE

In thermal equilibrium the currents are zero. Knowing this we can show that the fermi energy must be a constant in TE.

Focus on the electron current (looking at the hole current yields the same result):

\[ J_e(x) = qn_o(x)\mu_e F(x) + qD_e \frac{dn_o}{dx} \]

with \( n_o(x) = N_c e^{-(E_c-E_F)/kT} \)

First we calculate \( \frac{dn_o}{dx} \) and manipulate it a bit to get:

\[ \frac{dn_o}{dx} = \frac{n_o}{kT} \left( -\frac{dE_c}{dx} + \frac{dE_F}{dx} \right) = \frac{n_o}{kT} \left( -qF(x) + \frac{dE_F}{dx} \right) = \frac{\mu_e n_o}{D_e} \left( -F(x) + \frac{1}{q} \frac{dE_F}{dx} \right) \]

where we have used:

\[ \frac{dE_c}{dx} = -q \frac{d\phi}{dx} = qF(x) \quad \text{and} \quad \frac{\mu_e}{D_e} = \frac{q}{kT} \]

Putting this in \( J_e \) and simplifying the expression yields our final result:

\[ J_e(x) = \mu_e n_o(x) \frac{dE_F(x)}{dx} \]

From this result we see clearly that:

\[ J_e(x) = 0 \iff \frac{dE_F(x)}{dx} = 0 \]
Modeling non-equilibrium populations - quasi-Fermi levels

When we are not in thermal equilibrium, we can expect:

\[ n(x) \neq n_o(x) \quad \text{and} \quad p(x) \neq p_o(x) \]

because the populations are not necessarily in equilibrium with each other. However, it is likely that the electrons are in quasi-equilibrium with the conduction band states, and the holes are similarly in quasi-equilibrium with the valence band states. Thus the energy distribution of the electron population can be described by the Fermi function with the appropriate Fermi energy, now called the “electron quasi-Fermi level”, \( E_{Fn} \); the hole distribution can be similarly described using \( E_{fp} \). We find \( E_{Fn} \) and \( E_{Fp} \) knowing \( n \) and \( p \) by solving the following equalities for \( E_{Fn} \) and \( E_{Fp} \), respectively:

\[
  n(x) = \int_{E_c}^{\infty} \frac{\rho_{cb}(E)}{1 + e^{[E-E_{Fn}(x)]}} \, dE
\]

\[
  p(x) = \int_{-\infty}^{E_v} \frac{\rho_{vb}(E)}{1 + e^{[E-E_{Fp}(x)]}} \, dE
\]
Modeling non-equilibrium populations - quasi-Fermi levels, cont.

When the quasi-Fermi levels are within the band-gap by a few kT we can use the effective density of states concept to find analytical expressions for $E_{fn}$ and $E_{fp}$:

$$n(x) = \int_{E_c}^{\infty} \frac{\rho_{cb}(E)}{1 + e^{[E - E_{fn}(x)]}} dE \approx N_c e^{[E_c - E_{fn}(x)]} \quad \Rightarrow \quad E_{fn}(x) = E_c - kT \ln\left(\frac{n(x)}{N_c}\right)$$

$$p(x) = \int_{-\infty}^{E_v} \frac{\rho_{vb}(E)}{1 + e^{[E - E_{fp}(x)]}} dE \approx N_v e^{[E_{fp}(x) - E_v]} \quad \Rightarrow \quad E_{fp}(x) = E_v + kT \ln\left(\frac{p(x)}{N_v}\right)$$

Returning to our discussion of the electron and hole currents, we find that in non-equilibrium situations, where they are not necessarily zero, they are related to product of the carrier concentration, the mobility, and the gradient of in the quasi-Fermi levels:

$$J_e(x) = \mu_e n(x) \frac{dE_{fn}(x)}{dx} \quad \text{and} \quad J_h(x) = \mu_h p(x) \frac{dE_{fp}(x)}{dx}$$

Gradients in the quasi-Fermi levels act like net drift fields pushing the relevant carriers.
Quasi-Fermi levels - quasi-Fermi levels in some specific situations

A very important finding involving quasi-Fermi levels is that we can write the electron and hole currents in terms of the gradients of the respective quasi-Fermi levels, at least in the low field limit where drift mobility is a valid concept. We found:

\[ J_n(x) = \mu_e n(x) \frac{\partial E_{fn}(x)}{\partial x} \]

and

\[ J_p(x) = \mu_h p(x) \frac{\partial E_{fp}(x)}{\partial x} \]

Examples:

A. Uniformly doped n-type semiconductor with uniform E-field

At low to moderate E-fields, the populations are not disturbed from their equilibrium values, and we have

\[ n(x) \approx n_o \approx N_d \quad \text{and} \quad p(x) \approx p_o = n_i^2 / N_d \]

Also, \( E_{fn} \approx E_{fp} \approx E_f - q\phi(x) \), so:

\[ J_e \approx \mu_e n_o (-q \frac{\partial \phi}{\partial x}) = q\mu_e n_o F_x \quad \text{and} \quad J_e \approx q\mu_h p_o F_x \]

As expected, the currents are the respective drift currents.
B. **P-side of forward biased n⁺-p junction, long-base limit:**

Diode diffusion theory gives us \( n(x) \) on the p-side*:

\[
n(x) = n_{op} \left[ (e^{qv_{ab}/kT} - 1)e^{-x/L_e} + 1 \right], \text{ where } n_{op} = n_i^2/N_{Ap}
\]

When \( v_{AB} \gg kT \), and \( x \) is not many \( L_e \), we can approximate \( n(x) \) as:

\[
n(x) = n_{op} \left[ (e^{qv_{ab}/kT} - 1)e^{-x/L_e} + 1 \right] \approx n_{op} e^{qv_{ab}/kT} e^{-x/L_e}
\]

from which we find:

\[
E_{fn}(x) = E_c + kT \ln \left[ n(x)/N_c \right]
\]

\[
\approx E_c + kT \ln \left[ n_o/N_c \right] + qv_{ab} - kT \frac{x}{L_e}
\]

\[
\approx E_{fo} + qv_{ab} - kT \frac{x}{L_e}
\]

We see that \( E_{fn}(x) \) is \( qv_{AB} \) higher than the equilibrium Fermi level, \( E_{fo} \), at the edge of the depletion region on the p-side, and decreases linearly going away from the junction. Farther away from the junction, where \( x \) is many \( L_e \), \( n(x) \) approaches \( n_{op} \), and \( E_{fn}(x) \) approaches \( E_{fo} \).

Finally, notice that for low-level injection, \( p(x) \approx p_{po} \), and \( E_{fp} \approx E_{fo} \).
**Quasi-Fermi levels** - Illustrating examples A and B

Figure C-8 from Fonstad, *Microelectronic Devices and Circuits with quasi-Fermi levels added:*

**Example A:**

\[ E_{fn} \approx E_{fp} \approx E_{fo} \]

**Example B:**
Metal-Semiconductor Junctions - the structure

The structure is very simple

This is the "junction" we're talking about

but also very interesting, important, and useful
Metal-Semiconductor Junctions - barrier basics

- The evolution of the electrostatic barrier at the interface
  Initially we assume no surface states, i.e. bulk bands right to surface

- The energy band picture in isolation
  An isolated metal and an isolated semiconductor; neither "sees" the other

The vacuum reference levels are equal.
Both materials are neutral.
Note definitions of \( \phi \) (work function) and \( \chi \) (electron affinity)

\[
q\Phi_m = q\chi_s + kT \ln \left( \frac{N_C}{N_D} \right)
\]
Metal-Semiconductor Junctions - barrier basics

- The metal and semiconductor shorted electrically
  The short imposes a constant Fermi level throughout

The combined remains neutral, but the two materials become charged as electrons flow from the semiconductor to the metal until the Fermi levels are the same.

The semiconductor surface is slightly depleted at large separation; the depletion increases as they approach.
Metal-Semiconductor Junctions - barrier basics

- Shorted metal and semiconductor in physical contact

  As the distance between the metal and semiconductor decreases to zero, the depletion region grows

The final depletion region width is that needed to support a potential change equal to the built-in potential, $\phi_b (= \phi_m - \chi_s)$
The total structure is neutral, but there is now a dipole layer between the metal and semiconductor
To model this we use the depletion approximation
Metal-Semiconductor Junctions - progression of the barrier with decreasing separation
Metal-Semiconductor Junctions - barrier basics

- **Depletion approximation**

  The charge in the metal is approximated as a sheet (impulse) charge density at the surface, and charge in the semiconductor is approximated by a fully depleted layer $X_D$ wide:

\[
\begin{align*}
\rho(x) &\approx qN_D & \text{for} & \quad 0 < x \leq X_D \\
\rho(x) &\approx 0 & \text{for} & \quad X_D < x
\end{align*}
\]

Charge neutrality requires $Q^* = -qN_DX_D$

Remember we are dealing with sheet charge density, Coul/cm$^2$
• Depletion approximation (cont)

Integrating the charge divided by the dielectric constant yields the electric field

\[ E(x) = \int \frac{\rho(x)}{\varepsilon} \, dx \]

We get:

\[ E(x) \approx \begin{cases} 0 & \text{for } x \leq 0 \\ \frac{qN_D (x - X_D)}{\varepsilon} & \text{for } 0 < x \leq X_D \\ 0 & \text{for } X_D < x \end{cases} \]
Depletion approximation (cont)

Integrating the electric field yields the electrostatic potential

\[ \phi(x) = - \int E(x) \, dx \]

We get:

\[ \begin{align*}
\phi(x) & \approx qN_D(x - x_d)^2/2\epsilon & \text{for } x \leq 0 \\
0 & \text{for } 0 < x \leq x_d \\
\phi_b & \text{for } x_d < x
\end{align*} \]

Requiring that \( \phi(x) \) be continuous at \( x = 0 \) we find that the depletion region width, \( X_D \), must be

\[ X_D \approx \left( \frac{2\epsilon \phi_b}{qN_D} \right)^{1/2} \]

The profile is now fully determined.  

(i.e., we're done)
Real semiconductor surfaces - surface states

- Surface states
  There will be additional energy states on the surface of a semiconductor because the perfectly periodic lattice ends at the surface and many bonds are not "satisfied"
  These states... can have a very high density
  have a narrow distribution of energies within bandgap

- The energy bands in a semiconductor with surface states
  The surface states typically are sufficiently dense that in equilibrium the Fermi level falls within them at the surface and the surface is depleted:

\[ q\chi_s = f E_g - kT \ln(N_C/N_D) \]

Note: \( 0 < f < 1 \); for many III-V's \( f \approx 0.6-0.7 \)
Real semiconductor surfaces - surface states, cont.

• Estimating the number of surface states
  Unit cell 5.5Å by 5.5Å → 3 x 10^{14} cells/cm^2 at surface
  4 unsatisfied bonds per cell → ≈ 10^{15} states/cm^2
  If the states fall within 0.1 eV of each other → ≈ 10^{16} states/cm^2-eV
  This is very large!!

• What does this mean as a practical matter?
  Suppose φ_m - χ_s = 0.5 V, and that the effective separation of the charge in the surface states and metal is 25nm. The sheet charge density induced in this situation is:
  \[ Q^* = \varepsilon \Delta V/d = 10^{-12} \times 0.5 / 2.5 \times 10^{-6} = 2 \times 10^{-6} \text{ coul/cm}^2 \]
  The corresponding state density is \( Q^*/q \approx 10^{13} \text{ cm}^{-2} \)
  If all the surface states are active, the Fermi level at the surface will change only 1 mV; if only 10% are active it is only 10 mV. Only if 1%, or less, are active can the surface be unpinned.

• Conclusion
  The metal work function is often not the main determinant of the potential barrier in a metal-semiconductor junction.
Metal-Semiconductor Junctions - w. surface states

- The energy band picture in isolation with surface states

The surface of the semiconductor is depleted because of the charged surface states, independent of there being any metal nearby.

\[ q\Phi_m \]

\[ q\chi_s \]

\[ f \text{E}_g \]

\[ q\phi_{sd} = f \text{E}_g - kT \ln\left(\frac{N_C}{N_D}\right) \]

Note: \( 0 < f < 1; \) for many III-V's \( f \approx 0.6-0.7 \)
Metal-Semiconductor Junctions - w. surface states (cont.)

- Shorted metal and semiconductor, with surface states, in physical contact

When the density of surface states is high, as it typically is, the potential barrier that develops is dominated by the location of the surface states in the semiconductor band gap, rather than by the work function of the metal.

Otherwise, nothing is different and the same modeling holds.
Barrier heights

vs.

metal work function

-> the impact of surface states on metal-semiconductor barrier heights

Sze PSD Chap 8, Fig 7

-> the barrier height varies much less than does the work function of the metal

Sze PSD Chap 8, Fig 8
Applying bias to a metal-semiconductor junction

• What happens globally
  Potential step crossing junction changes
  Depletion region width and electric field change
  Current flows across junction

• Potential step change

![Diagram showing potential step change with reverse bias and forward bias]

Assuming all the bias appears across the junction, the potential barrier changes from $\phi_b$ to $\phi_b - V_{AB}$

$\phi_b \longrightarrow \phi_b - V_{AB}$

Note: Forward bias decreases the barrier
Reverse bias increases the barrier
Applying bias to a metal-semiconductor junction, cont.

- **Depletion region width and field changes**
  Wherever $\phi_b$ appears in the expressions for depletion region width and electric field, it is replaced by $\phi_b - v_{AB}$:

  **Depletion region width:**
  \[
  X_D \longrightarrow \left[\frac{2\varepsilon(\phi_b - v_{AB})}{qN_D}\right]^{1/2}
  \]
  **Note:** The depletion region width decreases in forward bias. Reverse bias increases the depletion region width.

  **Peak electric field:**
  \[
  E_{pk} = \left[\frac{2\varepsilon \phi_b qN_D}{\varepsilon}\right]^{1/2} \longrightarrow \left[\frac{2\varepsilon(\phi_b - v_{AB}) qN_D}{\varepsilon}\right]^{1/2}
  \]
  **Note:** The peak electric field decreases in forward bias. Reverse bias increases the field strength.

- **Note:** potential step and depletion region changes are the same as happens in a p-n junction
Applying bias to a metal-semiconductor junction, cont.

- **Currents**

  Note: the barrier seen by electrons in the metal does not change with bias, whereas the barrier seen by those in the semiconductor does.

  Thus the carrier flux (current) we focus on is that of majority carriers from the semiconductor flowing into the metal. Metal-semiconductor junctions are primarily majority carrier devices.

Minority carrier injection into the semiconductor can usually be neglected; more about this later.
Applying bias to a metal-semiconductor junction, cont.

- **Currents, cont.**

  The **net current** is the current from the semiconductor to the metal, minus the current from the metal to the semiconductor:

  \[
  i_D(v_{AB}) = i_{Dm\rightarrow s}(v_{AB}) - i_{Ds\rightarrow m}(v_{AB})
  \]

  **Semiconductor to metal,** \(i_{Ds\rightarrow m}(v_{AB})\)

  **Four factors:**
  1. \(N_{Dn} \exp \left[-\frac{q(\phi_b - v_{AB})}{kT}\right]\), the number of carriers that can cross the barrier, \((\phi_b - v_{AB})\)
  2. \(R\), the rate at which the carriers that can cross, get across
  3. \(A\), the cross-sectional area
  4. \(-q\), the charge per carrier

  \[
  i_{Ds\rightarrow m}(v_{AB}) = -q \ A \ R \ N_{Dn} \ \exp \left[-\frac{q(\phi_b - v_{AB})}{kT}\right]
  \]

  **Metal to semiconductor,** \(i_{Dm\rightarrow s}(v_{AB})\)

  **Not a function of voltage** (because barrier seen from metal doesn't change)

  **Must equal** \(i_{Ds\rightarrow m}(v_{AB})\) **when** \(v_{AB} = 0\), i.e. \(i_{Ds\rightarrow m}(0)\)

  \[
  i_{Dm\rightarrow s}(v_{AB}) = i_{Ds\rightarrow m}(0) = -q \ A \ R \ N_{Dn} \ \exp \left[-\frac{q\phi_b}{kT}\right]
  \]
Applying bias to a metal-semiconductor junction, cont.

- Currents, cont.
  
  Thus, the net current is:
  
  \[ i_D(v_{AB}) = q A R N_{Dn} \exp(-q\phi_b/kT) \left[ \exp(qv_{AB}/kT) - 1 \right] \]

  
  What we haven't done yet is say anything about \( R \) (at least not enough)
  The modeling meat is in \( R \)!

- Barrier transit rate models (models for \( R \))
  
  Different models assume that different factors are limiting the flow, and they result in different dependences of \( R \) (and thus of the \( i_D \)) on the device and material parameters and temperature.

  - **Thermionic emission theory** - the flow is limited by the rate at which carriers try to cross the barrier

  - **Drift-diffusion theory** - the flux is limited by the rate at which carriers cross the depletion region and reach the barrier

  - **Combination theories** - both of the above factors play a role and must be included in the modeling
Applying bias to a metal-semiconductor junction, cont.

- **Image force barrier lowering**

  An electron leaving a metal sees an image force pulling it back:

  \[
  \phi = \frac{q^2}{16\pi\varepsilon d}
  \]

  We see that the potential step at the surface of a metal is not abrupt as we have modeled it:

  This reduces the barrier seen by the carriers.  

  (next foil)
Applying bias to a metal-semiconductor junction, cont.

- Image force barrier lowering (cont.)

The image force reduces the barrier:

Furthermore the barrier reduction increases with increasing reverse bias:

This means the current does not saturate in reverse bias (unlike the case in a p-n diode).
Comparison of m-s junctions and p-n junctions

Lessons from i-v modeling results:

– Comparing metal to n-Si and p⁺-Si to n-Si diodes, i.e. same n-sides

• The m-s current is higher at the same bias \( i_{D,m-s}(v_{AB}) > i_{D,p-n}(v_{AB}) \) at same \( v_{AB} \)

• There is no minority carrier injection or storage in the m-s diode
  modulation and switching can be much faster

• The reverse bias, or "off" current of an m-s diode does not truly saturate
  turn-off is not has hard, but we can still have sharp breakdown and avalanche

The first two differences play major roles in several applications of m-s diodes
What metal-semiconductor junctions are good for

Note: The key features that make m-s junctions useful are…
- majority carrier devices, negligible minority carrier injection
- relatively low barrier to forward current flow
- depletion and field extend to surface

Important Applications

• Ohmic contacts
  an essential component of any electronic device

• Determining doping profiles
  a key diagnostic technique in device fabrication/processing

• Shunt diodes
  to reduce switching transients in bipolar transistor logic

• Microwave diodes
  another use taking advantage of negligible excess carrier injection

• FET gate (MESFETs)
  the subject of Lecture 4

• Ultraviolet detectors
  to be discussed in Lecture 22