Signals and Systems
Spring 2008
Lecture #24
(5/15/2008)

• CD Recording and Playback
• Course wrap-up

Brief History of Sound Reproduction
• Mechanical — phonograph (Thomas Edison 1877)
Brief History of Sound Reproduction (cont.)

- Mechanical — modern gramophone

- Magnetic — tape cassette (~1927—)

- Both the mechanical and analog magnetic recordings are analog — The recorded $CT$ signals are proportional to the original sound signals.
Digital Recording — Compact Disk (CD)

Digitally recorded medium with optical read-out.

What is on a CD?

- Protective layer (plus label)
- Reflective layer (typically aluminum)
- Polycarbonate (injection molded)
Read out from a CD

- From pits to bits — the length of the pits and lands represents a binary number

1001000100010001000100010000000001

3Δ 4Δ 4Δ 5Δ 3Δ 9Δ 4Δ

Feedback control to stay focused with a constant linear speed
Advantages of Digital Over Analog Recording

- A much better signal/noise ratio, — just like all other digital devices over their analog counterparts (*E.g.* Digital vs. analog wireless phones, HDTV vs. analog TV, *etc.*).

  *E.g.* For a standard 16-bit dynamic range of CD recording, assume the noise is mainly due to the quantization error in *LSB*, then the signal/noise ratio is:

  \[
  2^{16} \Rightarrow 20\log_{10}(2^{16}) \approx 96 \text{ dB!}
  \]

  \[
  \Rightarrow \frac{\text{Power of signal}}{\text{Power of noise}} \approx 10^{10} !
  \]

- Can perform error corrections to further improve fidelity of recording and playback

  *E.g.* Minor scratches will not affect the sound quality.
“Boundary Conditions” for Recording — Human Hearing

- Experimentally measured human hearing threshold as a function of frequency \( f = \omega/2\pi \).

![Graph showing hearing threshold vs. frequency]

The frequency range of human hearing is approximately 20Hz-20kHz

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Block Diagram of CD Recording

- A straightforward and simplistic approach — 1 × sampling

\[ f_s = 44.1 \text{ kHz} \]

\[ f_s = \omega_s/2\pi = 44.1 \text{ kHz} > 2 \times \text{maximum frequency of human hearing} \]

The specific value was chosen to synchronize with video

\[ f_s = (3 \text{ samples/line}) \times (490 \text{ lines/frame}) \times (30 \text{ frames/s}) = 44.1 \text{ kHz} \]
**Problem of 1x sampling:** Analog AAF with sharp cut-off difficult to implement.

**Problem of Analog AAF**

- In order to avoid aliasing, need an analog LPF with a very sharp cut-off. Specifically, need a 80-dB roll-off over the frequency range from 20 kHz to 24.1 kHz.

*E.g.* Frequency response of $n$th-order Butterworth filters

- Problem — Significant *nonlinear* phase distortion in the passband.
Solution: 4x Oversampling

Now the shifted spectra are far apart (176.4 kHz), the requirement on the front-end analog AAF is much more relaxed.

4x Oversampling (continued)

- Now the 80-dB attenuation can be easily achieved over the frequency range of 20-156.4 kHz, e.g., with a fifth-order Butterworth filter.
### DT Processing of Oversampled Signals ($\Omega = 2\pi \omega_s/\omega_s$)

- Once the signals are converted into DT signals, it is much easier to construct DT LPF with a very sharp cut-off.

**E.g.** A 200-point FIR filter with a linear phase.

![Frequency response](image1.png)

**Frequency response**

Linear phase in the passband

![Unit-sample response](image2.png)

**Unit-sample response**

### Now We Can Eliminate Signals Beyond Our Hearing Range

- Now we have solved the problem of aliasing, but ended up with $4\times$ amount of data needed for CT signal reconstruction. In the frequency domain, $\sim3/4$ space is empty.
- Solution — $4\times$ downsampling.
4× Downsampling — Decimation

Compressed by 4× in the time domain (speed up by a factor of 4).
This is what you have on a CD.

4× Downsampling — Frequency Domain

- Stretched by 4× in the frequency domain.
Decimation in the Frequency Domain
(Slide 19 in Lecture #16)

\[
X_b(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} x_b[k]e^{-j\omega k} \quad (x_b[k] = x_p[kN])
\]

\[
= \sum_{k=-\infty}^{+\infty} x_p[kN]e^{-j\omega k} \quad \text{Let } n = kN \text{ or } k = n / N
\]

\[
= \sum_{n \text{ a multiple of } N} x_p[n]e^{-j\omega (n/N)}
\]

\[
= \sum_{n=-\infty}^{+\infty} x_p[n]e^{-j(\omega/N)n} \quad \text{(since } x_p[n \neq kN] = 0)\]

\[
= X_p(e^{j(\omega/N)}) \quad \text{stretched in frequency by } N
\]

CD Playback — Reverse Process of Recording

4× Upsampling

Step 1: Inserting 3 zeros between two adjacent samples (need to slow down by a factor of 4).

Data stored on a CD \( y_b[n] \) → Insert 3 zeros → \( y_p[n] \)

\[
\cdots \bullet \cdots \bullet \cdots \bullet \cdots \bullet \cdots
\]

\( \omega \) in frequency by \( N \)
CD Playback — Reverse Process of Recording
4× Upsampling (continued)

Step 2: Pass $y_p[n]$ through a DT LPF — DT interpolating

\[ y_p[n] \rightarrow \text{DT LPF} \rightarrow y[n] \]

CD Playback (continued)

Step 3, DT to CT conversion.

\[ y[n] \rightarrow \text{D/C} \rightarrow x_p(t) \]

Sequence

Impulse train in time-domain
Finally, CT lowpass → get back the original CT signal

\[ x_p(t) \xrightarrow{\text{CT LPF}} x(t) \]

Wrap-up

- CD is said to be one of the most successful electronic instruments ever invented. AND 6.003 enables you to understand how it is recorded and played back, and many other goodies ...
Good luck with the final and have a nice summer!