Signals and Systems
Spring 2008

Lecture #1 (2/5/2008)
Prof. Qing Hu

(Slides thanks to D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

1) Administrative details
2) Signals

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6.003 Course Structure

- Weekly lectures (Tuesday and Thursday)
  — Introduce/derive main theorems and properties, examples and demos.

- Weekly recitations (Wednesday and Friday)
  — More detailed discussions and examples to reinforce the lecture materials.

- Weekly tutorials (Monday and Tuesday)
  — Help with homework.

- Office hours (TBD)

6.003 Texts

- Text book

  A. V. Oppenheim and A. S. Willsky, with S. H. Nawab,

  *Signals and Systems* (total 11 chapters)

- MATLAB text:

  J. R. Buck, M. M. Daniel, and A. C. Singer (BDS),

  *Computer Explorations in Signals and Systems Using MATLAB* (total 11 chapters)
6.003 Grading Scheme
(4-2-9)

- Quiz 1 (March 13) 20%
- Quiz 2 (April 17) 20%
- Final (May 19-23) 40%
- Problem sets* (10 + 1) and participation 20%

* Problem sets include both analytical parts and MATLAB exercises. Handed out on Thursdays and due the following Wednesday except the quiz week.

6.003 Policies

Collaboration Policy
- Discussion of concepts in homework is encouraged.
- Copying other people’s homework or codes is not permitted.

Deadlines
- Homework must be submitted in recitation on due date. Late submission will receive substantially reduced credit.

Homework Extension Policy
- Every student gets one extension. Must notify the his/her TA before midnight before the due date.
6.003 Syllabus

- Lectures 1-4: Introduction of signals and systems, LTI and convolution.
- Lectures 5-7: Fourier series, frequency response and filtering
- Lectures 8-12: Fourier transform
- Lectures 13-16: Sampling and modulation
- Lectures 17-19: Laplace transform
- Lectures 20-21: Feedback
- Lectures 22-23: z-transform
- Lecture 24: Fun lecture, music CD recording and playback

Signals and Systems

6.003 is about introducing mathematical techniques to analyze signals and synthesize systems which process signals.

- Signals are something that vary with “time”.
- Systems process input signals to produce output signals.

Examples of signals

- Electrical signals --- voltages and currents in a circuit
- Acoustic signals --- audio or speech signals (analog or digital)
- Video signals --- intensity variations in an image (e.g. a CAT scan)
- Biological signals --- sequence of bases in a gene
- In 6.003, we will treat \textit{noise} as unwanted signals

Signal Classification

Type of Independent Variable

Time is often the independent variable. Example: the electrical activity of the heart recorded with chest electrodes — the electrocardiogram (ECG or EKG).
The variables can also be spatial

In this example, the signal is the intensity as a function of the spatial variables $x$ and $y$.

Independent Variable Dimensionality

An independent variable can be 1-D ($t$ in the EKG) or 2-D ($x, y$ in an image).

In 6.003, focus on 1-D for mathematical simplicity but the results can be extended to 2-D or even higher dimensions. Also, we will use a generic time $t$ for the independent variable, whether it is time or space.
Continuous-time (CT) Signals

- Most of the signals in the physical world are CT signals, since the time scale is infinitesimally fine, so are the spatial scales. E.g. voltage & current, pressure, temperature, velocity, etc.

Discrete-time (DT) Signals

- $x[n]$, $n$ — integer, time varies discretely

- Examples of DT signals in nature:
  - DNA base sequence
  - Population of the $n$th generation of certain species

- Notation in 6.003: $x(t)$ — CT, $x[n]$ — DT
Many human-made Signals are DT

Ex.#1 Weekly Dow-Jones industrial average

Ex.#2 digital image

Ex#3. ticking clock

Why DT? — Can be processed by modern digital computers and digital signal processors (DSPs).

Signals with symmetry

- Periodic signals
  CT \[ x(t) = x(t + T) \]

\[ x(t) \]

\[ \cdots, -T, 0, T, 2T, \cdots \]

- Ex. 60-Hz power line, computer clock, etc.

DT \[ x[n] = x[n + N] \]

\[ x[n] \]

\[ \cdots, n \cdots \]
Signals with symmetry (continued)

- Even and odd signals
  
  **Even** \( x(t) = x(-t) \) or \( x[n] = x[-n] \)

  \[ x(t) = \begin{cases} x(-t) & \text{for even signals} \\ 0 & \text{for odd signals} \end{cases} \]

  **Odd** \( x(t) = -x(-t) \) or \( x[n] = -x[-n] \)

  or \( x(0) = 0 \)
  
  \[ x[n] = 0 \]

Signals with symmetry (continued)

- Any signals can be expressed as a sum of Even and Odd signals. That is:

  \[ x(t) = x_{\text{even}}(t) + x_{\text{odd}}(t) , \]

  where

  \[ x_{\text{even}}(t) = \frac{[x(t) + x(-t)]}{2} , \]

  \[ x_{\text{odd}}(t) = \frac{[x(t) - x(-t)]}{2} . \]

**Ex.** DT unit-step function.
Right- and Left-Sided Signals

A right-sided signal is zero for $t < T$ and a left-sided signal is zero for $t > T$, where $T$ can be positive or negative.

Bounded and Unbounded Signals

Whether the output signal of a system is bounded or unbounded determines the stability of the system.
Real and Complex Signals

In general, $x$ is a complex quantity and has:

- a real and imaginary part, or equivalently
- a magnitude and a phase angle.

We will use whichever form that is convenient.

A very important class of signals is complex exponentials:

- CT signals of the form $x(t) = e^{st}$
- DT signals of the form $x[n] = z^n$

where $z$ and $s$ are complex numbers.

For example, suppose $s = j\pi/8$ and $z = e^{j\pi/8}$, the exponentials are purely imaginary, then the real parts are

$$
\Re \{x(t) = e^{st}\} = \Re \{e^{j\pi t/8}\} = \cos(\pi t/8),
$$

$$
\Re \{x[n] = z^n\} = \Re \{e^{j\pi n/8}\} = \cos[\pi n/8].
$$
For example, suppose \( s = \sigma + j\omega \) for a CT signal

\[
\mathcal{R}\{x(t) = e^{st}\} = \mathcal{R}\{e^{(\sigma+j\omega)t}\} = e^{\sigma t} \cos(\omega t),
\]

\( \sigma > 0 \)

\( \sigma < 0 \)

For example, suppose \( z = e^{(\sigma+j\omega)} \), the exponential is complex for a DT signal, then the real part is

\[
\mathcal{R}\{x[n] = z^n\} = \mathcal{R}\{e^{(\sigma+j\omega)n}\} = e^{\sigma n} \cos(\omega n).
\]

\( \sigma > 0 \)

\( \sigma < 0 \)
Summary

• We are awash in a sea of signal — sounds, visual, electrical, thermal, mechanical, etc.

• Signals can be time-varying or spatially varying, can be a function of multiple variables. However, in 6.003, we will mostly deal with 1D signals and use a generic time $t$ to represent the variable, whether it is time, space, or something else.

• DT signals and systems become more and more important for signal processing, and will be a major part in 6.003.

Next lecture covers:
O & W pp. 38-56