Problem 1.1

(a) Rectangular: \( \frac{1+j}{\sqrt{3}+j} \cdot \frac{\sqrt{3}-j}{\sqrt{3}-j} = \frac{\sqrt{3}+1+j(\sqrt{3}-1)}{3+1} = \frac{(\sqrt{3}+1)+j(\sqrt{3}-1)}{4} \).

Polar: \( \frac{1+j}{\sqrt{3}+j} = \frac{\sqrt{2}e^{j\pi/12}}{2e^{j\pi/12}} = \frac{1}{\sqrt{2}}e^{j\pi/12} \).

You can verify that these are equivalent.

(b) \( e^{j} + e^{3j} = e^{2j}(2 \cos 1) \). So, the magnitude is \( 2 \cos 1 \) and the phase is 2.

(c) \( (\sqrt{3}-j)^8 = (2e^{-j\pi/6})^8 = 2^8e^{8(-j\pi/6)} = 256e^{-j\pi/3} = -128 + j128 \sqrt{3} \).

(d) \( \int_{0}^{\infty} e^{-2t} \cos(\pi t) \, dt = \int_{0}^{\infty} e^{-2t} (e^{j\pi t} + e^{-j\pi t}) \, dt = 2 \left( \int_{0}^{\infty} e^{(-2+j\pi)t} \, dt + \int_{0}^{\infty} e^{(-2-j\pi)t} \, dt \right) = 2 \left( \frac{1}{2+j\pi}e^{(-2+j\pi)t}|_{0}^{\infty} + \frac{1}{2-j\pi}e^{(-2-j\pi)t}|_{0}^{\infty} \right) = \frac{2}{4+\pi^2} \).

(e) Let \( r \) be the magnitude of \( z \) and \( \theta \) be the phase of \( z \). Then, \( \frac{1-z^n}{1-z} = \sum_{k=0}^{n-1} z^k = \sum_{k=0}^{n-1} (re^{j\theta})^k \).

So, \( Re \left\{ \frac{1-z^n}{1-z} \right\} = Re \left\{ \sum_{k=0}^{n-1} r^k e^{jk\theta} \right\} = \sum_{k=0}^{n-1} r^k \cos(k\theta) \).

(f) There are often several ways of manipulating complex numbers, even for the simplest cases. Knowing which method solves a particular problem is easiest or most insightful is a skill that would be useful to acquire in the course of learning 6.003.

Example 1.2

All three methods produce the same result:

\( x(-2t + 6) \)

Problem 1.3

(a) Periodic. The period is \( \frac{2\pi}{3\pi/2} = \frac{4}{3} \).

(b) Periodic. We have \( 2\pi(3) = \frac{3\pi}{2}(4) \), and 3 and 4 share no common factors, so the period is 4.

(c) Periodic. The period is \( \text{LCM}(4/3, 6) = 12 \).
(d) Periodic. The period is LCM(4, 6) = 12.

(e) Not periodic. 4/3 and 2π are incommensurate numbers, i.e. their ratio is irrational.

(f) Not periodic. There exist no integers m and N such that 3N = 2πm.

(g) Periodic. Even though each signal has fundamental period 2, the fundamental period of x_g(t) is 1.

Problem 1.4

(a) \(x[n] = -\delta[n + 1] + 2\delta[n - 1] - \delta[n - 2]\).

(b) \(x[n] = -u[n + 1] + u[n] + 2u[n - 1] - 3u[n - 2] + u[n - 3]\).

Problem 1.5

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Problem 1.6

(a) True.

(b) True.

(c) True.

(d) False. Let system 1 do \(x(t) \rightarrow y(t)\) and system 2 do \(y(t) \rightarrow z(t)\). If \(y(t) = e^{j\pi t}x(t)\) and \(z(t) = e^{-j\pi t}y(t)\), we see that each system is time-varying, but the overall system is \(z(t) = x(t)\), which is time-invariant.