Directions: The exam consists of 5 problems on pages 2 to 19. Please make sure you have all the pages. Enter all your work and your answers directly in the spaces provided on the printed pages of this booklet. Please make sure your name is on all sheets. DO IT NOW! All sketches must be adequately labeled. Unless indicated otherwise, answers must be derived or explained, not just simply written down. This examination is closed book, but students may use one 8 1/2 × 11 sheet of paper for reference. Calculators may not be used. Note that the problems are not in the order of difficulty. Solve the problems that you can first.

NAME: 

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<th>Check your section</th>
<th>Section</th>
<th>Time</th>
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<td>Yajun Fang</td>
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<table>
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<th>Problem</th>
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<th>Grader</th>
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Problem 1 (12 Points)

Consider the following three systems:

- **System A:**
  \[ y(t) = \text{Re}\{x(t)\} \]

- **System B:**
  \[ y(t) = x(t) + \frac{dx(t + 1)}{dt} \]

- **System C:**
  \[ y[n] = e^{j\omega_0(n+1)}x[n] \quad \omega_0 \neq 0 \]

where \( x \) and \( y \) are the input and output of each system, respectively.

Circle YES or NO for each of the following questions for each of these systems (no explanation is required).

<table>
<thead>
<tr>
<th></th>
<th>System A</th>
<th>System B</th>
<th>System C</th>
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</thead>
<tbody>
<tr>
<td>Is the system linear?</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
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<tr>
<td>Is the system time invariant?</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
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<tr>
<td>Is the system causal?</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Is the system stable?</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
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Work Page
Problem 2  (20 Points)

The impulse response of an LTI system is given by

\[ h(t) \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \]

(a) Compute and sketch below the step response \( s(t) \) of the system (i.e. the output when the input is the unit step \( u(t) \)).

\[ s(t) \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \]
(b) One cycle of a sine wave $x(t)$ (shown below) is the input to the system. Compute and sketch the corresponding output $y(t)$. 

\[ y(t) \]
Problem 3  (21 Points)

Consider the signal $x[n]$ shown below:

(a) What is the maximum number of independent Fourier series coefficients that this signal can have?

Maximum number =

Brief explanation:

(b) Derive the Fourier series coefficients $a_k$ of $x[n]$. 
Work Page
(c) On the graph below, plot the phase of $a_k$. 
Problem 4 (21 Points)

For each of the following systems, determine whether an LTI system exists that produces $y(t)$ for the input $x(t)$.

(a)

$$x(t) = 5 + \cos(\pi t + \frac{\pi}{4}) \quad \rightarrow \quad A \quad \rightarrow \quad y(t) = \cos^2(\frac{\pi}{2} t)$$

System A CAN / CANNOT be an LTI system. (circle the right answer)

**Brief explanation:**

(b)

$$x(t) \quad \rightarrow \quad B \quad \rightarrow \quad y(t)$$

Where $x(t), y(t)$ are periodic signals with fundamental periods $T$ and $\frac{T}{2}$, respectively.

System B CAN / CANNOT be an LTI system. (circle the right answer)

**Brief explanation:**
Where \( x_1(t), x_2(t) \) and \( y(t) \) are periodic signals with fundamental period \( T \). The Fourier series coefficients \( a_k \) of \( x_1(t) \) and \( b_k \) of \( x_2(t) \) as well as one period of \( y(t) \) are given below.
System C CAN / CANNOT be an LTI system. (circle the right answer)

Brief explanation:
Problem 5 (26 Points)

Suppose we have an LTI system with an impulse response $h[n]$

(a) Find the frequency response $H(e^{j\omega})$ of this system.

$$H(e^{j\omega}) =$$

(b) Given the following input signal $x_b[n]$, find out the output signal $y_b[n]$.

$$x_b[n] = 1$$

$$y_b[n] =$$

(c) Given the following input signal $x_c[n]$, find out the output signal $y_c[n]$.

$$x_c[n] = (-1)^n$$

$$y_c[n] =$$
(d) Consider the following system:

\[
x[n] \rightarrow h[n] \rightarrow y_1[n] \rightarrow h[n] \rightarrow y_2[n] \rightarrow \ldots \rightarrow y_{M-1}[n] \rightarrow h[n] \rightarrow y_M[n]
\]

Given the following input signal \( x[n] \):

\[
\begin{array}{cccccccc}
-4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\vdots & 7 & 7 & 7 & 7 & 7 & 7 & 7 & \ldots & \vdots
\end{array}
\]

Evaluate this expression: \( \lim_{M \to +\infty} y_M[n] \).

\[
\lim_{M \to +\infty} y_M[n] =
\]

(e) For a DT periodic signal \( x[n] \) with arbitrary period \( N \), evaluate \( \lim_{M \to +\infty} y_M[n] \).

\[
\lim_{M \to +\infty} y_M[n] =
\]