Directions:

- Please make sure your name is on all sheets.
- Unless indicated otherwise, answers must be derived or briefly explained. All sketches must be adequately labeled.
- Enter all your work and answers directly in the spaces provided on the printed pages of this booklet. Additional work spaces are supplied from Pages 24 through 26. These pages will not be graded unless you specify that you are continuing a particular problem on a particular continuation page.
- This Quiz is closed book except for two 8 1/2 × 11-inch sheet of paper (four sides).
- Tables of Fourier series and transform properties and pairs are a separate handout. You must turn in your transform sheets when you turn in your quiz.
- The bluebook is for your scratch work. We will not grade anything in your bluebook.
- No other materials and aids, such as calculators, cell phones, or music players, are permitted.

NAME:

<table>
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<td>Stephen Hou</td>
<td>Chun-Chieh Lin</td>
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Please leave the rest of this page blank for use by the graders:

<table>
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<th>Score</th>
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PROBLEM 1 (19%)

For a signal \( x(t) \), the magnitude and phase of its Fourier transform, \( X(j\omega) \), are given below.

A. Compute \( \int_{-\infty}^{\infty} x(t) dt \) and \( \int_{-\infty}^{\infty} |x^2(t)| dt \):

\[
\int_{-\infty}^{\infty} x(t) dt = \text{__________}
\]

\[
\int_{-\infty}^{\infty} |x^2(t)| dt = \text{__________}
\]
Work page for problem 1
B. For each of the signals below, find the matching magnitude and phase plots from the ones on the following two pages. **No explanation necessary. No partial credit.**

<table>
<thead>
<tr>
<th>Signal</th>
<th>Magnitude Plot</th>
<th>Phase Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x(-t) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{dx(t)}{dt} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x(t)e^{j2t} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x(t) * \frac{\sin(3t)}{3t} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x(t)[\delta(t) + \delta(t - 8)] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x(t) \cos(t) )</td>
<td></td>
<td>XXX</td>
</tr>
<tr>
<td>( x(t - \frac{\pi}{10}) )</td>
<td></td>
<td></td>
</tr>
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</table>

†: Please note that this function is \( x(cos(t)) \), not \( x(t)cos(t) \). We are not asking you to choose a phase plot for this signal.

**Magnitude plots for Part B.**

![Magnitude plots](image)
Phase plots for Part B.

I

II

III

IV

V

VI

VII
PROBLEM 2 (22%)

In this problem we will consider anti-causal DT LTI filters.

A. Write down the linear, constant coefficient difference equation (LCCDE) for the system with unit sample (impulse) response as given by $h_1[n]$.

LCCDE:  

B. Derive the frequency response of the system described by $h_1[n]$.

$$H_1(e^{j\omega}) =$$
C. How does the magnitude and phase of this frequency response differ from the magnitude and phase of the frequency response of the causal DT filter described by $h_2[n]$ (shown below)?

Circle one of the following. No explanation necessary. No partial credit.

1. $|H_1(e^{j\omega})| = |H_2(e^{j\omega})|
2. $|H_1(e^{j\omega})| = 2|H_2(e^{j\omega})|
3. $|H_1(e^{j\omega})| = |H_2(e^{j\omega})| + 1$
4. $|H_1(e^{j\omega})| = \frac{1}{|H_2(e^{j\omega})|}$

Circle one of the following. No explanation necessary. No partial credit.

1. $\angle H_1(e^{j\omega}) = \angle H_2(e^{j\omega})$
2. $\angle H_1(e^{j\omega}) = 2\angle H_2(e^{j\omega})$
3. $\angle H_1(e^{j\omega}) = -\angle H_2(e^{j\omega})$
4. $\angle H_1(e^{j\omega}) = \angle H_2(e^{j\omega}) + \pi / 2$
D. Consider the DT LTI filter described by the unit sample (impulse) response:

\[ h_3[n] = \begin{cases} 
(-\alpha)^{-n} & n \leq 0 \\
0 & n > 0 
\end{cases} , \]

where \(0 < \alpha < 1\). Write the linear constant coefficient difference equation that describes this system \textbf{without using infinite sums}.

\[ LCCDE: \]

E. Derive the frequency response for the system described by \( h_3[n] \).

\[ H_3(e^{j\omega}) = \]

F. Which type of filter is implemented by the system \( h_3[n] \)? \textbf{Circle one of the following. No explanation necessary. No partial credit.}

1. Low pass
2. High pass
3. Band pass
Work page for problem 2
PROBLEM 3 (20%)

Consider the following CT system:

\[
x(t) \xrightarrow{H_c(j\omega)} y(t)
\]

and the following DT processing system:

\[
x(t) \xrightarrow{\text{Impulse to Sequence}} x_d[n] \xrightarrow{H_d(e^{j\Omega})} y_d[n] \xrightarrow{\text{Sequence to Impulse}} y_p(t) \xrightarrow{T_s} y_c(t)
\]

where \( p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s). \) We would like to design \( H_d(e^{j\Omega}) \) to perform the same processing as \( H_c(j\omega) \) for the input

\[x(t) = \frac{\sin(50\pi t)}{\pi t}.
\]
A. Sketch $Y(j\omega)$, the output of the CT system for the input described above. Label your $\omega$ and $Y(j\omega)$ axes with values.

B. Sketch $X_d(e^{j\Omega})$, the DTFT of $x_d[n]$, for $T_s = \frac{1}{100}$. Label your axes with values.
C. Design $H_d(e^{j\Omega})$ as an ideal DT high-pass filter so that the complete DT processing system implements $H_c(j\omega)$ [i.e., $y_c(t) = y(t)$ for this $x(t)$] for $T_s = \frac{1}{100}$. Sketch your $H_d(e^{j\Omega})$ on the axes below. Label your axes with values.

\[
\begin{array}{c}
H_d(e^{j\Omega}) \\
\hline
\Omega
\end{array}
\]

D. What happens if you use the exact $H_d(e^{j\Omega})$ that you designed in part C with $T_s = \frac{1}{50}$? Sketch and label the output, $Y_c(j\omega)$, of the DT processed signal with this new sampling rate with the specified $x(t)$.

\[
\begin{array}{c}
Y_c(j\omega) \\
\hline
\omega
\end{array}
\]
E. What is the upper limit, if one exists, of $T_s$ for which you can construct an ideal DT high-pass filter for $H_d(e^{j\Omega})$ so that $y_c(t) = y(t)$ for this $x(t)$? Write a value or write “No Solution”.

$T_s < \underline{\phantom{000}}$

F. What is the upper limit, if one exists, of $T_s$ for which you can construct an ideal DT high-pass filter for $H_d(e^{j\Omega})$ so that $y_c(t) = y(t)$ if the input, $x(t)$, is changed to the signal drawn here:

![Signal Diagram]

Write a value or write “No Solution”.

$T_s < \underline{\phantom{000}}$
Work page for problem 3
PROBLEM 4 (21%)

Consider the following CT LTI system

\[ H(j\omega) = \frac{(100 - j\omega)}{10(1+j\omega)} \]

A. Determine \( h(t) \), the impulse response of the system.

\[ h(t) = \]__

B. Determine a linear constant coefficient differential equation (LCCDE) described by this frequency response.

LCCDE:__________________
C. Sketch approximate Bode plots of the magnitude and phase responses of this system, $|H(j\omega)|$ and $\angle H(j\omega)$, on the axes below. Only straight-line approximations are needed. Label all important features (e.g. slope of each line and points where lines intersect).
D. Determine $y(t)$ when $x(t) = 3e^{-t}u(t)$.

$$y(t) = \underline{\hspace{3cm}}$$

E. Determine $y(t)$ when $x(t) = 4\cos(50t + \frac{\pi}{4})$.

$$y(t) = \underline{\hspace{3cm}}$$
Work page for problem 4
PROBLEM 5 (18%)

Consider a DT signal $x[n]$, about which we have the following information:

- $x[n]$ is real and even
- $X(e^{j\omega}) = 0$ for $\omega_M \leq |\omega| \leq \pi$

A. Which of the following graphs could represent $X(e^{j\omega})$? Circle the letters for all possible graphs. No explanation is necessary. No partial credit.
B. We are given this signal with one sample corrupted, i.e. we are actually given

\[ x_1[n] = \begin{cases} 
  x[n] & \text{for all } n \neq n_0 \\
  x[n_0] + C & \text{if } n = n_0 
\end{cases} \]

for some real \( C \neq 0 \) and integer \( n_0 \). We are also given \( X_1(e^{j\omega}) \), the DTFT of \( x_1[n] \). Our ultimate goal is to restore the original signal, i.e. to determine \( n_0 \) and \( x[n_0] \). We will do this in several steps.

(i) First, write a closed-form expression for \( x_1[n] \) in terms of \( x[n] \) and a unit sample (impulse) function.

\[ x_1[n] = \] ________________

(ii) Next, write \( X_1(e^{j\omega}) \) in terms of \( X(e^{j\omega}) \) and the quantities \( \omega_M, n_0, \) and \( C \). Also plot \( |X_1(e^{j\omega})| \) and \( \angle X_1(e^{j\omega}) \) only for \( \omega_M < |\omega| < \pi \) on the axes below. Fully label your plot.

\[ X_1(e^{j\omega}) = \] ________________
\[ \angle X_1(e^{j\omega}) \]
(iii) Give a valid expression for $C$ in terms of $X_1(e^{j\omega})$ and $\omega_M$.

$$C = \phantom{\text{expression}}$$

(iv) Give a valid expression for $n_0$ in terms of $X_1(e^{j\omega})$ and $\omega_M$.

$$n_0 = \phantom{\text{expression}}$$

(v) Therefore, what is $x[n_0]$, in terms of the received signal $x_1[n]$ and its Fourier transform $X_1(e^{j\omega})$?

$$x[n_0] = \phantom{\text{expression}}$$
Work Page for Problem 5
Additional Work Page
Additional Work Page