Quiz Date: Thursday, April 17, 2008
Time: 7:30–9:30 pm
Location: Rooms 32-144 & 32-155
Coverage: Chapters 1–7 of O&W through Section 7.4, Lectures and Recitations through April 4, Problem Sets #1–6, and that part of Problem Set #7 involving problems from Chapter 7.
Notes: The quiz is closed book except for two 8.5” × 11” two-sided sheets of notes. No calculators, cell phones, or music players are allowed. We will provide copies of Tables that have been posted online under the Quiz 2 section of the course website.

Marathon Office Hours: The TAs will hold office hours from 2–8 p.m. on Wednesday, April 16 and again from 10 a.m.–3 p.m. on Thursday, April 17. A schedule will be posted on course website.

Quiz Review Session: The quiz review session will be on Tuesday, April 15, 7:30–9:30 p.m. in 34-101.

Practice Problems: The attached set of problems should provide you with ample opportunity to exercise your understanding of and facility with the material covered on this quiz. These problems will be covered in the quiz review session. The solutions to these problems will be posted on the 6.003 website on the evening of Tuesday, April 15. Copies of Quiz 2 from previous semesters are available online. Both of these sets of problems should help to spark questions you might want to discuss with the TAs during their office hours.
Problem #1  The purpose of this problem is to test your understanding of the continuous-time Fourier transform.

Consider the system depicted below,

\[ x(t) \rightarrow \bigotimes h(t) \rightarrow y(t) \]

where

\[ x(t) = \frac{\sin(4\pi t)}{\pi t} \]
\[ p(t) = 2\cos(2\pi t) \]

and the impulse response \( h(t) \) is given by

\[ h(t) = 1 + 3\sin(4\pi t) + 2\cos(8\pi t) \].

(a) Provide a labeled sketch of \( R(j\omega) \), the Fourier transform of \( r(t) \).

(b) Determine \( y(t) \).

Problem #2  The purpose of this problem is to test your understanding of the continuous-time Fourier transform.

Consider the signal

\[ x(t) = \cos(2\pi t) + \sin(6\pi t) \].

Suppose that this signal is the input to each of the LTI systems with impulse responses given below. Determine the output in each case.

(a) \( h(t) = \frac{\sin(4\pi t)}{\pi t} \)
Problem #3  The purpose of this problem is to test your understanding of the discrete-time Fourier transform.

We want to design a discrete-time, linear, time-invariant system that has the following property:

If the input is

\[ x[n] = \left( \frac{1}{2} \right)^n u[n] - \frac{1}{4} \left( \frac{1}{2} \right)^{n-1} u[n - 1], \]

then the output is

\[ y[n] = \left( \frac{1}{3} \right)^n u[n]. \]

(a) Find the impulse and frequency responses of a discrete-time, LTI system that has this property.

(b) Find a difference equation relating \( x[n] \) and \( y[n] \) that realizes the desired frequency response.

Problem #4  The purpose of this problem is to test your understanding of the discrete-time Fourier transform.

The signal \( x[n] \) is shown below. Note that \( x[n] = 0 \) for \( n \leq 1 \) and \( n \geq 7 \).

Let \( X(e^{j\omega}) \) denote the discrete-time Fourier transform of \( x[n] \).

(a) Evaluate \( X(e^{j\omega}) \) for \( \omega = 0 \).
(b) We can write

\[ X(e^{j\omega}) = A(e^{j\omega})e^{j\phi(\omega)}, \]

where \( A(e^{j\omega}) \) is real (possibly positive or negative) and \( \phi(\omega) \) is some function of \( \omega \). Determine the function \( \phi(\omega) \).

(c) Evaluate

\[ \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{dX(e^{j\omega})}{d\omega} \right|^2 d\omega. \]

(d) We can write

\[ X(e^{j\omega}) = \text{Re} \{ X(e^{j\omega}) \} + j \text{Im} \{ X(e^{j\omega}) \}. \]

Let \( x_r[n] \) be the signal whose Fourier transform is \( \text{Re} \{ X(e^{j\omega}) \} \), and let \( x_i[n] \) be the signal whose Fourier transform is \( j \text{Im} \{ X(e^{j\omega}) \} \). Determine the signals \( x_r[n] \) and \( x_i[n] \).

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**Problem #5**  The purpose of this problem is to test your understanding of Bode plots.

Consider a system with frequency response

\[ H(j\omega) = \frac{\left(\frac{j\omega}{10} + 1\right)^2}{(j\omega + 1)\left(\frac{j\omega}{100} + 1\right)}. \]

Sketch the Bode plot for \( |H(j\omega)| \) and \( \angle H(j\omega) \), using the straight line approximation. Make sure to label your sketch.

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**Problem #6**  The purpose of this problem is to test your understanding of sampling.

Reactionary Systems, Inc., has proposed the following hybrid scheme to overcome certain computer speed limitations affecting the realization of discrete-time LTI filters. They suggest (see the diagram below) converting the incoming discrete-time sequence \( x[n] \) into a continuous-time impulse train

\[ x(t) = \sum_{n=-\infty}^{\infty} x[n] \delta(t - nT), \]

which is then filtered by a continuous-time system with impulse response \( h(t) \), to yield

\[ y(t) = h(t) * x(t). \]

The output is finally sampled and reconverted to a discrete-time sequence \( y[n] = y(nT) \).
(a) Find the overall response of this system to the input $x[n] = \delta[n]$. Expressions for the clock signals are given by $s_1(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ and $s_2(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$. Assume some arbitrary shape for $h(t)$ and illustrate your argument with sketches of $x(t)$, $y(t)$, and $y[n]$.

(b) This system contains other non-constant inputs in addition to $x[n]$ (the clock signals) and includes subsystems that are not exactly LTI (the DT/CT converters). Nevertheless, it functions overall as a linear, time-invariant discrete-time system. Justify this statement and give a formula for the discrete-time unit-sample response $h[n]$.

(c) Does the system remain LTI if the phases of the clock signals are different, e.g. if

$s_1(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ and $s_2(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT - \Delta)$? If it does remain LTI, determine the unit sample response (if it is different from (b)).

(d) Repeat part (c) if the periods of the clock signals are different, e.g. if $s_1(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_1)$ and $s_2(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_2)$, where $T_1 \neq T_2$.

(e) For the original clock signals, find the discrete-time frequency response

$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$.

HINT: Note that

$\sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} = \int_{-\infty}^{\infty} \left[ \sum_{n=-\infty}^{\infty} h[n] \delta(t - nT) \right] e^{-j\omega t} dt,$

and that

$\sum_{n=-\infty}^{\infty} h[n] \delta(t - nT) = h(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$.
Problem #7  The purpose of this problem is to test your understanding of the continuous-time Fourier transform and sampling.

(a) Draw an accurately labeled sketch of $R(j\omega)$, the Fourier transform of $r(t)$. (HINT: consider $r(t)$ as the difference of two simple impulse trains of different periods.)

(b) For a fixed value of $T$ find the maximum value of $W$ and the value of $K$ in the diagram below such that $y(t) = x(t)$. 

\[
\begin{align*}
    &\text{\(x(t)\)} & &\text{\(x(t) \cdot r(t)\)} & &\text{\(y(t)\)} \\
    \downarrow &   & \times & & \downarrow & \\
    \text{\(X(j\omega)\)} &   & r(t) & & H(j\omega) & y(t) \end{align*}
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