1. For the following exercises on complex numbers, assume $j = \sqrt{-1}$.

   a. Write simplified expressions for $jj$, $jjj$, and $jjjj$.

   b. Draw a labeled plot of vector $z$ where

   $$z = 1 + j2$$

   and the x-axis and y-axis correspond to the real and imaginary components, respectively. Be sure to include labels for the magnitude and phase (in degrees as opposed to radians) of $z$.

   c. Calculate the magnitude, $K$, and phase (in radians), $\Phi$, of $z$, where

   $$z = Ke^{j\Phi} = 1 + j2$$

   d. Calculate the real component, $a$, and imaginary component, $b$, for

   $$a + jb = 6e^{j\pi/6}$$

2. For the following exercises on Fourier Series, use only the complex exponential form of the Fourier Series:

   $$x(t) = \sum_{n=-\infty}^{\infty} \hat{X}_n e^{j\omega nt}$$

   $$\hat{X}_n = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-j\omega nt} dt$$

   a. Calculate the Fourier Series $\hat{X}_n$ for

   $$x(t) = \sin(\omega t) + 2\cos(3\omega t)$$

   b. Calculate the Fourier Series $\hat{X}_n$ for the periodic waveform shown in Figure 1 below

   i. Express $\hat{X}_n$ in terms of its real and imaginary components

   $$\hat{X}_n = A_n + jB_n$$
ii. Express $\hat{X}_n$ in terms of its magnitude and phase components

$$\hat{X}_n = |\hat{X}_n| e^{j\phi_n}$$

iii. Plot the magnitude of $\hat{X}_n$ over the index range of $n = -5$ to $n = 5$ assuming $T_p = T/4$.

3. For the following exercises on the Fourier Transform, assume the Fourier Transform definition given in class:

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

a. Calculate the Fourier Transform $X(f)$ for the non-periodic waveform shown in Figure 2 below

i. Express $X(f)$ in terms of its real and imaginary components

$$X(f) = A(f) + jB(f)$$

ii. Express $X(f)$ in terms of its magnitude and phase components

$$X(f) = |X(f)| e^{j\phi(f)}$$

iii. Plot the magnitude $X(f)$ of over a reasonable frequency range to see its key characteristics. Assume that $A = 1$ and $T_p = 1$. 

Figure 1: Periodic pulse waveform.

Figure 2: Non-periodic pulse waveform.
4. Consider the Fourier Series of signal $x(t)$ which is shown in Figure 3. Assuming $x(t)$ is periodic with period $T$, plot the Fourier Series (i.e., $A_n$ and $B_n$) of $x(t-T/2)$.

![Figure 3](image)

5. For each signal shown in Figure 4 below, circle your choice of whether the Fourier Series or Fourier Transform is appropriate for frequency domain analysis, and then circle whether the frequency domain description will be purely real, purely imaginary, or complex.

<table>
<thead>
<tr>
<th>$x(t)$:</th>
<th>Fourier Series</th>
<th>Fourier Transform</th>
<th>real</th>
<th>imaginary</th>
<th>complex</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(t)$:</td>
<td>Fourier Series</td>
<td>Fourier Transform</td>
<td>real</td>
<td>imaginary</td>
<td>complex</td>
</tr>
<tr>
<td>$z(t)$:</td>
<td>Fourier Series</td>
<td>Fourier Transform</td>
<td>real</td>
<td>imaginary</td>
<td>complex</td>
</tr>
</tbody>
</table>

![Figure 4](image)