1. In this problem, we will explore the two-stage demodulator shown in Figure 1 below. This system resembles the receiver we have used in labs. In particular, the USRP board can be thought of as providing the first stage of mixing and filtering in hardware, and then Matlab of providing the second stage of mixing and filtering software.

**Figure 1**

a) Given the definition of the Fourier Transform, namely

\[
x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df \quad \text{and} \quad X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt
\]

show that shifting in frequency corresponds to multiplication in time by a complex exponential:

\[
e^{-j2\pi f_{\text{shift}}t}x(t) \leftrightarrow X(f + f_{\text{shift}})
\]

b) Given the Fourier Transform property in part (a), plot \(X_1(f)\) and \(X_2(f)\) assuming that \(r(t) = e^{j2\pi (10 \text{MHz})t}\).
c) Given your results from part (b), what is an appropriate signal $z(t)$ such that the desired signal (and only the desired signal) appears at the demodulator output, $y(t)$? Justify your response by plotting $X_3(f)$ and $Y(f)$ for your choice of $z(t)$.

d) Now suppose that $r(t)$ was instead chosen to be $r(t) = \cos(2\pi(10 \text{MHz})t)$. Is there an appropriate choice for $z(t)$ that would allow the desired signal (and only the desired signal) to appear at the output, $y(t)$? Justify your answer with pictures.

2. Consider the image rejection mixer shown in Figure 2 along with associated signals and lowpass filter response. For the questions to follow, please be sure to label plots with key frequency and amplitude values.

![Image rejection mixer diagram](image.png)

(a) Plot $A(j2\pi f)$ and $B(j2\pi f)$
(b) Plot $C(j2\pi f)$ and $D(j2\pi f)$
(c) Plot $E(j2\pi f)$ and $G(j2\pi f)$
(d) Plot $Y(j2\pi f)$
(e) Why do you think they call this structure an image rejection mixer?

3. Reflections from the walls and other objects in the room often cause several delayed and attenuated versions of the transmitted signal to be received at the receiver as shown in Figure 3. You have experienced this in the lab when you noticed that in certain locations your boards had very poor reception, on certain tones. This “multi-path” effect can be modeled as a filter connecting the transmitter and the receiver. If the delay of the signal on the “line-of-sight” path is $T_1$ and attenuation $A_1$, and the delay of the reflected signal is $T_2$ and amplitude $A_2$, write the expression for the frequency response of the “channel filter” $H_{\text{Channel}}(f)$, and plot the frequency response for the following values $A_1=0.1$, $T_1=10\text{ns}$ and $A_2=0.05$, $T_2=30\text{ns}$.
4. Consider the block diagram of the continuous-time filter shown in Figure 4, whose behavior is described by the differential equation:

\[ 100y(t) + 20 \frac{d}{dt}y(t) + \frac{d^2}{dt^2}y(t) = 1000x(t) \]

Figure 4. Continuous-time filter

(a) Determine \( H_{\text{filter}}(f) \), i.e., the filter response from \( x(t) \) to \( y(t) \).

(b) Determine the output, \( y(t) \), given the input
i. \( x(t) = \sin(2\pi(0.1\text{Hz})t) \)
ii. \( x(t) = \sin(2\pi(10\text{Hz})t) \)
iii. \( x(t) = \sin(2\pi(1000\text{Hz})t) \)

(c) Plot the magnitude of \( H_{\text{filter}}(f) \) versus \( f \) using Matlab. Be sure to label the magnitude at the frequency points determined in part (b).

(d) Is the filter response lowpass, highpass, or bandpass?

(e) Discretize the continuous-time filter using the sampling period \( T_s \) and determine coefficient vectors \( a \) and \( b \) for the Matlab filter command, to realize the discretized filter. Apply the discrete filter to the sinewaves in (a)i, (a)ii, and (a)iii, for \( T_s=0.1\text{ms} \). How far can you increase the sampling period \( T_s \) while not affecting the filter performance?