This problem set has six questions, some with multiple parts. Answer them as clearly and concisely as possible. Turn in your solutions during class on Friday, May 2, 2008.

1  A Little TDM

Alyssa P. Hacker is running some experiments on a TDM circuit-switched network switch. All packets are of the same size. The rate of the link connected to the switch is $\mu = 1000$ packets per second. The link can support 24 concurrent conversations between different pairs of nodes.

In a given experiment, she picks a random number of conversations, $r$, and has them communicate using the switch. She measures the expected per-frame delay at the receiver ($D$), and also monitors the number of frames in the switch with time (if a frame shows up earlier than its allocated time slot, the switch stores the frame until it is ready to be sent). The expected number of frames in the switch is $N$. No frames are dropped in her experiments.

Somewhat to her surprise, she finds that the product $D \cdot \mu$ is often not equal to $N$. Explain this “violation” of Little’s law.

2  Slotted Aloha

We derived the formula for the throughput of a broadcast network using Aloha in class. That version of Aloha was unslotted because each sender was allowed to send at any time. Suppose we modify the Aloha protocol so that time is now divided into timeslots of duration $\tau$ seconds, where $\tau$ is the time to transmit a given packet (all packets are the same size). A sender is allowed to transmit only at the start of a timeslot.

A. Calculate the formula for the throughput, $R$, of this variant of Aloha, called slotted Aloha.

B. What is the maximum value of $R$?

C. How does the maximum throughput for unslotted Aloha compare with the maximum throughput for slotted Aloha?

3  Probabilistic Channel Access

Ben Bitdiddle invents a probabilistic channel access scheme. Here, time is “slotted” as in slotted ALOHA and the length of each time slot is the time taken to send a single link-layer frame. All frames are of the same size. Each node sends frames with a certain probability, $p$. If two or more nodes send frames in the same slot, we will assume that they collide and no frame will be received
successfully; otherwise, any transmitted frame will be received correctly. There are \( N \) nodes all in broadcast range of each other. Assume that each node always has data to send. Answer the following questions, justifying each answer.

A. What is the channel utilization of Ben’s protocol?

B. What value of \( p \) maximizes the channel utilization of Ben’s protocol?

C. Suppose \( N \) is large. How does the maximum possible channel utilization of Ben’s protocol compare with the maximum possible channel utilization of slotted ALOHA?

4 Smart Movie Backups

Over many months, you and your friends have painstakingly collected a 1,000 Gigabytes (aka 1 Terabyte) worth of movies on computers in your dorm (we won’t ask where the movies came from). To avoid losing it, you’d like to back the data up on to a computer belonging to one of your friends in New York.

You have two options:

A. Send the data over the Internet to the computer in New York. The data rate for transmitting information across the Internet from your dorm to New York is 1 Megabyte per second.

B. Copy the data over to a set of disks, which you can do at 100 Megabytes per second (thank you, firewire!). Then rely on the US Postal Service to send the disks by mail, which takes 7 days.

Which of these two options (A or B) is faster? And by how much?

5 A Little Queueing

You send a stream of packets of size 1000 bytes each across a network path from Cambridge to Berkeley. You find that the one-way delay varies between 50 ms (in the absence of any queueing) and 125 ms (full queue), with an average of 75 ms. The transmission rate at the sender is 1 Mbit/s; the receiver gets packets at the same rate without any packet loss.

A. What is the mean number of packets in the queue at the bottleneck link along the path (assume that any queueing happens at just one switch).

You now increase the transmission rate to 2 Mbits/s. You find that the receiver gets packets at a rate of 1.6 Mbits/s. The average queue length does not change appreciably from before.

B. What is the packet loss rate at the switch?

C. What is the average one-way delay now?
6 From Bit Errors to Packet Errors

When running experiments in the lab over a communication link of rate $\mu$ bits/s, you find that the bit-error rate between sender and receiver is $p$. The sender takes a group of $S$ payload bits, then attaches a header of size $H$ bits, then sends the resulting frame to the receiver. The sender also attaches an error detection code (a CRC or checksum) to the end of the packet.

The receiver calculates the CRC for each received frame and drops any frame whose CRC doesn’t match. If the CRC matches, the frame is considered to be “good”. Assume that any frame with a bit error will have a CRC that does not match, and any frame with a CRC that matches does not have any errors.

At the link-layer, let’s define the communication throughput as the number of payload bits per second from good frames that the receiver is able to obtain.

A. Write an expression for the throughput of the link in terms of the parameters defined above.

B. What should you set the payload size $S$ to, in terms of $p$ and $H$, to maximize the throughput of the link? Assume that $p << 1/H$ and use suitable approximations to simplify your answer; for example, for $x << 1$, $\ln(1 + x) \approx x$, $(1 + x)^n \approx 1 + nx$, etc.)