Problem 1: Define the following words, phrases and symbols.

1. Finite state machine, finite automaton
2. Sipser page 49. Determinism vs Nondeterminism
3. Sipser page 54. DFA vs NFA
4. Regular Language
5. Sipser page 53. \((Q, \Sigma, \delta, q_0, F)\)
6. \(\emptyset\)
7. \(\varepsilon\)
8. Epsilon Transition
10. Sipser page 36, 40. A machines can accept many strings, but only a single language. To avoid confusion, we will usually say a machine accepts a string and recognizes a language.

Problem 2: Are the following statements true or false?

1. It is possible for a finite automaton to recognize an infinite language.
2. Every deterministic finite automaton is also a nondeterministic finite automaton.
3. For every nondeterministic finite automaton, there is an equivalent nondeterministic finite automaton that has no epsilon transitions.
4. For every nondeterministic finite automaton, there is an equivalent nondeterministic finite automaton that has a single accept state.
5. If you swap the accept states and the reject states on ANY finite automaton, the new machine will recognize the complement of the original language.
6. The class of languages recognized by non-deterministic finite automata is closed under complementation.
7. The class of languages recognized by non-deterministic finite automata is not closed under set difference.
8. Sipser problem 1.24. For any string \(w = w_1 w_2 \ldots w_n\), the reverse of \(w\), written \(w^R\), is the string \(w\) in reverse order, \(w_n \ldots w_2 w_1\). For any language \(A\) let \(A^R = \{w^R | w \in A\}\). If \(A\) is regular, then so is \(A^R\).
Problem 3: Create finite automata for each of the following languages over the alphabet $\Sigma = \{0, 1\}$. Give a characterization for each state.

1. The language of strings that do not contain an odd number of 1s.
2. The language of strings that contain '0110' and end in '1'.
3. The language of strings that contain a '0' and don’t end in '10'.
4. The language of part 2 concatenated with that of part 3.

Problem 4: Let’s prove that the following automaton recognizes exactly the language $L = \{w \in \{0, 1\}^* | w$ contains a 10 and ends in 1$\}$. To do this, we will need to prove that our FA (1) accepts all strings in $L$ and (2) does not accept any string not in $L$.

1. Characterize each state.
2. Forward direction (accepts all strings in $L$). Proof by Induction?
3. Reverse direction (does not accept any string outside of $L$). Proof by Contradiction?

Problem 5: Show that the language $L_5 = \{s \in \{0, 1\}^* \mid s$ divisible by 5$\}$ is regular.

Problem 6: Optional (if we have enough time)
(Closure of regular languages under perfect shuffle) (Sipser 1.41)
For languages $A$ and $B$, let the perfect shuffle of $A$ and $B$ be the language:

$$\{ w \mid w = a_1b_1a_2b_2 \ldots a_kb_k, \text{ where } a_1a_2\ldots a_k \in A, b_1b_2\ldots b_k \in B. \}$$

Show that regular languages are closed under the perfect shuffle operation.

Problem 7: Optional (if we have enough time)
An all-paths-NFA $M$ is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if and only if every possible state that $M$ could be in after reading $x$ is a state from $F$. Prove that all-NFAs recognize exactly the regular languages. (Notice the contrast with NFAs)