Problem 1: Let’s review the following new terms and concepts.

1. BPP: Bounded-error Probabilistic Polynomial Time, \( \epsilon \leq 1/3 \).
2. RP: one-sided error, error only on inputs in the language
3. Branching Program: A DAG with a variable query at each node and only two leaves, labeled 0 and 1. Each internal node has two outgoing edges labeled 0 and 1. There is also a start node.

Problem 2: (Sipser 7.27)
A coloring of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Let

\[
3\text{COLOR} = \{ (G) | \text{the nodes of } G \text{ can be colored with three colors such that no two nodes joined by an edge have the same color} \}.
\]

Show that \(3\text{COLOR}\) is NP-complete.

Problem 3: (Sipser 7.26)
You are given a box and a collection of cards as indicated in the following figure. Because of the pegs in the box and the notches in the cards, each card will fit in the box in either of two ways. Each card contains two columns of holes, some of which may not be punched out. The puzzle is solved by placing all the cards in the box so as to completely cover the bottom of the box, (i.e., every hole position is blocked by at least one card that has no hole there.) Let \(\text{PUZZLE} = \{ (c_1, \ldots, c_k) | \text{each } c_i \text{ represents a card and this collection of cards has a solution} \}\). Show that \(\text{PUZZLE}\) is NP-complete.

Problem 4: (Branching program equivalence test)
1. Give a read-once branching program $B_1$ that computes the function of three Boolean variables, $x_1$, $x_2$, and $x_3$, that has value 1 if and only if exactly one or exactly three of the variables have value 1.

2. Give a different read-once branching program $B_2$ that computes the same function as in part (a).

3. Compute the polynomials $p_1$ and $p_2$ associated with the output 1 box for programs $B_1$ and $B_2$, respectively, using the rules given in Sipser’s book, p. 378.

4. Choose arbitrary values from $Z_7$ for the three variables, and evaluate $p_1$ and $p_2$ to check that they indeed give the same result.