7. Undecidability of PCP
We showed MPCP, in which the input is a set of tiles + designated initial tile, is undecidable.

By reducing $A$ to MPCP.

Q: Why doesn't the given reduction reduce $A$ to PCP?
A: Has trivial matches, e.g., $[a]$. 

So show PCP is undecidable, reduce MPCP to PCP, that is, show
if PCP is decidable so is MPCP.

So decide MPCP
Suppose we are given $P = \{[t_1^1], [t_1^2], \ldots, [t_1^k]\}$ = set of
$[d] = [t_1]$ = initial tile.

We want to know if $\exists$ match beginning with $[t_1^1]$.

Construct an instance of ordinary PCP that has a match (beg.
with any tile) iff $P$ has a match beginning with $[t_1^1]$.
(Can't just use the same instance + initial - there could be some match, but no match starting with initial tile.)

Construction (technical):
Add 2 new symbols, * and $\Box$.
If $u = u_1 \ldots u_n$, then define $\ast u = \ast u_1 \ast u_2 \ldots \ast u_n$
$\underline{u} = u_1 \ast u_2 \ast \ldots \ast u_n$
$\ast \underline{u} = \ast u_1 \ast u_2 \ast \ldots \ast u_n$

Instance $P'$ of PCP:
$\{[\ast t_1], [t_1], [\ast t_2], \ldots, [\ast t_k], [\ast \Box]\}$