Lecture 16.75 - 17.25 - Dielectric Waveguides - Outline

• Refraction and diffraction
  Directing and guiding light

• Dielectric optics (guiding, confining, manipulating light)
  Cylindrical waveguides - (wires for light)
    Concept
    Properties
    Fabrication
  Slab waveguides
    Theory
    Lessons from modeling
  Rectangular waveguides
    Modeling
    Bends/Curves - loss vs. R
    Compact structures - Tees, Splitters
  Coupled rectangular waveguide structures
    Couplers
    Add/drop filters, Switches
Dielectric waveguides: Concept

- Total reflection at a dielectric interface

\[ \theta_c = \sin^{-1} \left( \frac{n_1}{n_2} \right) \]

\[ \theta_o = \sin^{-1} \left[ ( \frac{n_2}{n_1} ) \sin \theta_i \right] \]

- Light can be guided in a dielectric slab or cylinder of bounded by a lower index dielectric
**Dielectric waveguides: Structures**

- The guide cross-section can have any shape, but as a practical matter they are typically cylindrical or rectangular.

Cylindrical: ![Cylindrical Waveguide](image)

Rectangular: ![Rectangular Waveguide](image)

- Also, in practice at least three dielectrics are involved: a core, one or more cladding materials, and the ambient (i.e. air).

Air, $n_0$

**Core**

Note: The cladding shields the reflecting interface from the ambient and keeps it clean, etc. It is essential!

Fonstad/Palacios, 4/9,11/09  
Lecture 17 - Slide 3
Cylindrical dielectric waveguides

- Glass and plastic fibers

Some important characteristics of glass fibers

Loss spectrum of a plastic fiber

Apparatus for pulling a glass fiber from a preform

Figure 5-12  Spectral attenuation for all-glass fibers. (a) Multimode fiber (Corning Glass Works). (b) Single-mode fiber (Celanese Corporation).

Figure 5-14  Spectral attenuation for an all-plastic fiber cable (Mitsubishi Rayon America, Inc.).
Slab dielectric waveguides

- When the core dimensions are on the order of the wavelength, then discrete modes of propagation exist.

Example mode patterns and charts for symmetric and asymmetric slabs

Symmetric slab, $n_1 = 3.5$, $n_2 = 3.55$ (GaAs, AlGaAs)

Asymmetric slab, $n_1 = 2.29$, $n_2 = 1.5$, $n_0 = 1.0$ (Si$_3$N$_4$, SiO$_2$, Air)

(from Palais)
We will look for TE waves traveling in the z-direction, i.e. only $E_y$. We find that

$$H_y = H_x = 0 \quad \text{and} \quad E_y(x,z,T) \quad \text{[no y dep.]}
$$

$E_y$ must satisfy:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_0 \varepsilon_i \frac{\partial^2 E_y}{\partial t^2}
$$

We can write $E_y$ as:

$$E_y(x,z,T) = X(x) \cdot Z(z) \cdot T(t)
$$

Note: "i" is an index indicating the various regions in the x-direction.
Slab dielectric waveguides: Theory, cont.

- We have:
  \[ E_y(x, z, T) = X(x) \cdot Z(z) \cdot T(t) \]

  We next note that we want to have waves propagating, and this implies a lot about the z and t dependences, and also about what we want for the x variation. To have a wave traveling down the guide we will must have:

  \[ T(t) = \text{Re}[A e^{-j\omega t}] \]
  \[ Z(z) = \text{Re}[e^{j\beta_j z}] \]

  Putting these in \( E_y \) and putting \( E_y \) into the differential equation yields:

  \[ \frac{\partial^2 X_m(x)}{\partial x^2} - \left( \beta_j^2 - \mu_{\varepsilon} \varepsilon_i \omega^2 \right) X_m(x) = 0 \]

  We know the frequency of the signal, \( \omega \), and the only unknown is \( \beta_j \). We find it by matching the solutions to this differential equations in the various regions at the boundaries. The solutions are of the form:

  \[ X_m(x) = A_{\pm} e^{\pm \sqrt{\beta_j^2 - \mu_{\varepsilon} \varepsilon_i \omega^2} x} \]

  Note: "j" is an index corresponding to the different modes (if any).
Slab dielectric waveguides: Theory, cont.

- Before proceeding, note the following:
  \[ \mu_0 \varepsilon_i = \mu_0 \varepsilon_o \varepsilon_{ri} \quad \text{where} \quad \varepsilon_{ri} = n_i^2 \quad \text{and} \quad \mu_0 \varepsilon_o = 1/c^2 \]

  Thus, we can write
  \[ \mu_0 \varepsilon_i \omega^2 = \mu_0 \varepsilon_o n_i^2 \omega^2 = n_i^2 \frac{\omega^2}{c^2} = n_i^2 k_o^2 \]

  where \( k_o \equiv \omega/c = 2\pi/\lambda_o \), and \( \lambda_o \) is the free space wavelength.

- The velocity of light in a uniform medium with an index of refraction \( n_i \), is \( c/n_i \), and the phase velocity of the \( j \)th mode in a waveguide is \( \omega/\beta_j \). It is convenient to define an effective index of refraction for this waveguide mode, \( n_{eff,mj} \), as the ratio of its phase velocity to the velocity of light in free space:
  \[ n_{eff,mj} = \frac{c \beta_j}{\omega} = \frac{\beta_j}{k_o} \]

- We thus can write the factor in the second term of differential equation for \( X(x) \) any of several equivalent ways:
  \[ (\mu_0 \varepsilon_i \omega^2 - \beta_j^2) = (n_i^2 k_o^2 - \beta_j^2) = k_o^2 \left( n_i^2 - n_{eff,mj}^2 \right) \]
Slab dielectric waveguides

- Reviewing where we are so far, for the j-th TE mode of the slab, the electric field is given by:

\[ E_{y,j}(x,z,t) = X_j(x) \text{Re} \left[ e^{-j(\beta_j z - \omega t)} \right] \]

In this equation:
- \( \omega \): frequency/energy of the light: \( 2\pi\omega = \nu = c/\lambda_o \)
- \( \lambda_o \): free space wavelength
- \( \beta_j \): propagation constant of the j-th mode
- \( X_j(x) \): mode profile normal to the slab, obtained from either

\[ \frac{d^2 X_j}{dx^2} + \left( n_i^2 k_o^2 - \beta_j^2 \right) X_j = 0 \quad \Rightarrow \quad X_j(x) = A_j e^{\pm j \sqrt{(n_i^2 k_o^2 - \beta_j^2)} x} \]

or, equivalently,

\[ \frac{d^2 X_j}{dx^2} + k_o^2 \left( n_i^2 - n_{\text{eff},mj}^2 \right) X_j = 0 \quad \Rightarrow \quad X_j(x) = A_j e^{\pm j k_o \sqrt{(n_i^2 - n_{\text{eff},mj}^2)} x} \]

where
- \( k_o \): propagation constant in free space, \( = 2\pi/\lambda_o \)
- \( n_i \): refractive index in region i
- \( n_{\text{eff},mj} \): effective refractive index of the j-th waveguide mode
Slab dielectric waveguides

• In approaching a slab waveguide problem, we typically take the dimensions and indices in the various regions and the free space wavelength or frequency of the light as the "givens." The unknown is $\beta$, the propagation constant in the slab.

• We find that there are only discrete values for $\beta$ that yield a solution for $X(x)$. Each value corresponds to a mode of the slab.

• For a guided solution we want solutions with $\beta_m$'s such that

$$\left(n_i^2 k_o^2 - \beta_j^2\right) \leq 0$$

in the regions above and below the slab, and

$$\left(n_i^2 k_o^2 - \beta_j^2\right) \geq 0$$

in the slab.

• If these conditions are satisfied, $X_m(x)$ will be an exponential function decaying away for the slab in the outer regions and a sinusoidal function in the slab, and the mode will be guided.
Slab dielectric waveguides

- We can gain some interesting insight if we look at a simple symmetrical three-layer slab guide with the following layers:
  - Layer 1 - upper cladding, \( n = n_1 \)
  - Layer 2 - guiding layer, \( n = n_2 \)
  - Layer 3 - lower cladding, \( n = n_1 \)

- A bit of thinking about the requirement on the \( \beta_j \)'s stated in the previous foil leads us to the conclusion that we can only have guiding if the following relationship is true:

\[
\frac{n_1 k}{\lambda} \leq \beta_j \leq \frac{n_2 k}{\lambda}
\]

In terms of the effective mode index, this is equivalent to saying that \( n_{\text{eff}, m_j} \) must fall between \( n_1 \) and \( n_2 \).

\[
n_1 \leq n_{\text{eff}, m_j} \leq n_2
\]

- This "proves" our initial argument about how light is guided in slab guides.

**Note:** If the top and bottom cladding layers have different indices, they must both be less than \( n_2 \), and \( n_{\text{eff}, m_j} \) falls between \( n_2 \) and the larger of the two cladding indices.
Rectangular dielectric waveguides - modeling issues

- It is in general not possible to get a closed-form analytical solution to the field problem in an arbitrary rectangular guide, and an interactive computer solution must be done:

\[
\begin{array}{ccc}
  n_1 & n_2 & n_3 \\
  n_4 & n_5 & n_6 \\
  n_7 & n_8 & n_9 \\
\end{array}
\]

- A realistic guide, however, will typically be built on a slab and will have horizontal symmetry.

\[
\begin{array}{ccc}
  n_1 & n_2 & n_1 \\
  n_3 & n_4 & n_3 \\
\end{array}
\]

- Still, the problem is difficult to analyze.
Rectangular dielectric waveguides - modeling issues

- One route to a more manageable problem is to ignore the corner regions:

```
  n_2  n_1  n_3  n_2
```

In this case solutions can be found if \( n_1 = n_4 \), but this is a very restrictive condition.

- The most common and intuitively helpful approximate solution is what is called the "effective index" method. In this approximation we decompose the problem into three slab guide problems, first, two vertical slab guides, and, finally, one horizontal slab problem.

(continued on next slide)
Rectangular dielectric waveguides - Effective index method

- We look at the rectangular guide as being formed from two different slab guides, and calculate the effective indices for modes propagating in each:

\[
\begin{array}{c}
\text{n}_1 \quad \text{n}_2 \quad \text{n}_1 \\
\text{t} & \text{n}_3 & \text{n}_4 & \text{n}_3 \\
\text{n}_5 & & & \\
\end{array}
\]

- Vertical slab 1:

\[
\begin{array}{c}
\text{n}_1 \quad \text{n}_1 \\
\text{t} & \text{n}_3 & \text{n}_5 \\
\text{n}_5 & & \\
\end{array}
\]

\[n_{\text{eff},v1}\]

- Vertical slab 2:

\[
\begin{array}{c}
\text{n}_2 \quad \text{n}_2 \\
\text{t} & \text{n}_4 & \text{n}_5 \\
\text{n}_5 & & \\
\end{array}
\]

\[n_{\text{eff},v2}\]
Rectangular dielectric waveguides - Effective index method, cont.

• Then we build our third horizontal slab from these "layers", bounding layer 2 on either side by layers 1, and for each layer we use the appropriate effective refractive index:

```
| n_{eff,v1} | n_{eff,v2} | n_{eff,v1} |
```

• This is a symmetric slab guide and the solutions for this geometry are well known and fully tabulated*. The same is true of the solutions for asymmetric slab guides (the vertical problems on the previous slide)


• This method gives usefully accurate results, and is extremely intuitive and helpful when thinking about real guides
Single-mode rectangular waveguides

- Using the effective index method, the issue of determining how wide a rectangular guide can be and still be single mode, is reduced to finding how wide a symmetrical slab guide can be made before it will support a second-order mode.

- To do this we recognize that the second order mode will have a zero at the center of the slab and can thus be describe by a sin x function, as shown below (the center of the slab is \( x = 0 \)).

\[
X(x) = \begin{cases} 
-\sin\left[k_o \sqrt{n_2^2 - n_{\text{eff},m2}^2} \frac{W}{2}\right] e^{k_o \sqrt{n_{\text{eff},m2}^2 - n_1^2}(x+W/2)} & \text{for } x \leq -W/2 \\
\sin\left[k_o \sqrt{n_2^2 - n_{\text{eff},m2}^2} x\right] & \text{for } -W/2 \leq x \leq W/2 \\
\sin\left[k_o \sqrt{n_2^2 - n_{\text{eff},m2}^2} \frac{W}{2}\right] e^{-k_o \sqrt{n_{\text{eff},m2}^2 - n_1^2}(x-W/2)} & \text{for } W/2 \leq x
\end{cases}
\]

Fonstad/Palacios, 4/9/11/09
Single-mode rectangular waveguides, cont.

- We have already assured that the electric field is continuous across the boundaries by the way we have written the magnitude of the exponential terms for \(|x| > w/2\). By matching the slopes at \(x = \pm w/2\) we obtain an expression relating \(w\) and \(\beta_2\).

- We find:

\[
\lambda_o \cdot \frac{1}{2n_2} = \frac{\lambda_o}{2n_2} \cdot \frac{1}{\sqrt{1 - n_1^2/n_2^2}}
\]

As the second mode approaches cut-off it extends more and more into the cladding and the effective index of the mode approaches \(n_1\). Thus setting \(n_{eff,m2} = n_1\) will give the critical maximum width above which the guide will no longer be single mode. We find:

\[
\lambda_o \cdot \sqrt{n_2^2 - n_{eff,m2}^2} = 2 \tan^{-1} \left( \frac{\sqrt{n_2^2 - n_{eff,m2}^2}}{\sqrt{n_2^2 - n_{eff,m2}^2 - n_1^2}} \right)
\]

- Notice that the larger the difference between \(n_1\) and \(n_2\), the smaller \(w_{max}\) is.
Rectangular dielectric waveguides - Common structures

- **Rib guide:**

- **Buried rib guide:**

- **Ridge guide:**

* The buried rib and the ridge guides are very commonly and successfully used with in-plane laser diodes, as we shall see in Lecture 20.
Rectangular dielectric waveguides - Additional structures

• Implanted or diffused guide:

• Dielectric stripe:

• Slab coupled guide*:

* Note that the slab coupled guide is not a simple rectangular guide, but is instead a rectangular guide coupled to a slab guide. It is designed so that the higher order modes under the stripe couple to the modes of the slab, and are thus quite lossy. Only the lowest order mode under the stripe is a low loss mode. This feature is exploited a class of laser diodes, as we shall see in Lecture 20.
Rectangular dielectric waveguides - useful structures

- Numerous useful structures and devices can be built from rectangular dielectric waveguides:
  - Passive structures
    - Bends
    - Corners
    - Splitters/combiners
    - Couplers
    - In-line filters
    - Add-drop filters
  - Active structures
    - Switches
    - Modulators

- We will look at each of these, some more extensively than others (and the active devices more later on)
Rectangular dielectric waveguides - **bends**

- If a rectangular dielectric waveguide curves there must be some radiation loss, as seen on the right:

- The goal is to make the curve gradual enough so the loss is small. In doing this there is a direct trade-off between the level of confinement ($\Delta n$) and the bending radius. We find:

$$\alpha_{\text{curve}} \approx C_1 e^{-C_2 R}$$

The constants are related to the waveguide structure as:

$$C_1 = \frac{\cos^2 \left(k_{xg} w\right) \lambda_o e^{2k_{sl} w}}{4k_{xL} n_L w^2 \left[w + \frac{1}{2k_{xg}} \sin(2wk_{xg}) + \frac{1}{k_{xL}} \cos^2 \left(wk_{xg}\right)\right]}, \quad C_2 = 2k_{xL} \left(1 - \frac{\lambda_o \beta}{2\pi n_L}\right)$$

(continued on the next slide)
Rectangular dielectric waveguides - bends, cont.

• In these equations the following quantities appear:
  \( \beta \): z-directed propagation constant for guide
  \( k_{xg} \): x-directed propagation constant in guide
  \( k_{xL} \): decay constant in regions on outside of guide
  \( 2w \): width of guide
  \( n_g \): refractive index in guide
  \( n_L \): refractive index outside of guide

• Because of the difficulty of making sharp bends, optical waveguide layouts tend to look more like railroad switch yards, rather than city streets and intersections.
Rectangular dielectric waveguides - edge roughness loss

- Tighter confinement of the light to the guide, i.e., larger $\Delta n$, means there is less bending loss for a given curve radius,
  but it also means
  1. single modes guides have to be narrower
  2. the guides will be more susceptible to edge scattering losses


$$\alpha_{edge} \approx \frac{\cos^3 \theta}{2w_{eff} \sin \theta} \left( \frac{4\pi n_{eff} \Delta w}{\lambda_o} \right)^2$$

If we are modeling side-wall roughness the parameters are:
- $n_{eff}$: effective index of the mode in the guide
- $w_{eff}$: effective width of the guide, defined as $w + 2k_{xL}^{-1}$
- $\theta$: angle of incidence of the mode on the boundary

If are interested in the loss due to substrate and surface roughness, the width is replaced by the thickness, and $t_{eff}$ is $t + k_{y\text{sub}}^{-1} + k_{y\text{surf}}^{-1}$. 

Fonstad/Palacios, 4/9,11/09
Slab dielectric waveguides

- The issue of the trade-offs between bend radius, scattering loss, guide dimension, and index step
Achieving compact rectangular waveguide layouts

- Bends

Calculated transmission:

a. 30%

b. 60%

c. 98.5%

d. 98%

Achieving compact rectangular waveguide layouts

- **Resonator made using new 90° corners** - example fabricated using polysilicon on silicon dioxide

Q of resonance indicates 0.3 dB loss per corner

Rectangular dielectric waveguides - splitters, combiners

- Another useful structure is one where a signal is split into two signals, typically of equal strength

Notice that the guide widens before the split to become multi-mode. Lowest order mode widens, and the energy in it must be coupled into the much narrower lowest order modes of the two branch waveguides. This is an obvious recipe for loss.

It is possible to make the splitting loss from becoming too significant if the angle between the branches, $\phi$, is kept small:

$$\phi < \cos^{-1}\left(\frac{n_L}{n_g}\right) - \tan^{-1}\left(\frac{k_{xg}}{\beta}\right)$$

Achieving compact rectangular waveguide layouts

- Splitting Tees

Calculated transmission:

a. 30% (15%/side)

b. 99% (>49%/side)

Another method of splitting a signal into two parts, as well as to make switches, is to use what is called a waveguide coupler. This structure exploits the fact that the light is not totally confined to the waveguides:

When the waveguides are in close proximity, they must be viewed not as two separate guides with interacting modes, but as a composite structure with two basis modes.

Coldren and Corzine, Figures 7.29, 6.9b, and 6.21.
Rectangular dielectric waveguides - waveguide couplers

- There are two lowest order modes for the combined pair of guides, and they travel down the guide with slightly different velocities.

\[ L_{3\text{dB}} = \frac{\pi}{2\kappa}, \text{ where } \kappa = \frac{2k_{xg}^2 k_{xL} e^{-k_{xL}s}}{2\beta w (k_{xL}^2 + k_{xg}^2)} \]

Zappe, p. 209

The result is that over distance the energy will oscillate back and forth between the two guides with a period of \(4L_{3\text{dB}}\):
Planar waveguide integrated optics components

- Resonant ring couplers and channel dropping filters

Then

More Recently


Resonant ring couplers and channel dropping filters, cont.

Now

http://eecs.web.mit.edu April 09

Work reported at 2009 Photonics West, Jan. '09

Fonstad/Palacios, 4/9,11/09
Resonant ring couplers and channel dropping filters, cont.

Silicon waveguides and integrated photonic structures fabricated on a commercial silicon foundry fabrication line during standard CMOS IC processing (ref on slide 35).
Resonant ring couplers and channel dropping filters, cont.

Mach-Zender interference modulator

Phase shifter end-on cross-section
Resonant ring couplers and channel dropping filters, cont.

Representative two-stage ring oscillator

Si waveguide cross-section


Fabrication sequence
Note: Pictured dots are not yet embedded
Research done at UMass-Lowell

PL spectra for various diameter QDs
Note: Measurements on dots that are not embedded.

Fit to theory

* Latest work adds vapor transport embedding.