Lenses

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Important question

• Why is this toy so expensive
  – EF 70-200mm f/2.8L IS USM
  – $1700

• Why is it better than this toy?
  – EF 70-300mm f/4-5.6 IS USM
  – $550

• Why is it so complicated?

• What do these buzzwords and acronyms mean?
Lens quality varies!

source: the luminous landscape

Tuesday, April 28, 2009
source: the luminous landscape

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Center is usually OK

- [http://www.photo.net/equipment/canon/70-300do_2/](http://www.photo.net/equipment/canon/70-300do_2/)

*250x500 pixel crops, centre of frame f5.6*
Image corners are often sacrificed

- [http://www.photo.net/equipment/canon/70-300do_2/](http://www.photo.net/equipment/canon/70-300do_2/)

250x500 pixel crops, corner of frame f5.6
Max aperture is tough

- [http://www.photo.net/equipment/canon/70-300do_2/](http://www.photo.net/equipment/canon/70-300do_2/)

250x500 pixel crops, centre of frame f5.6
Gets better when stopped down

- http://www.photo.net/equipment/canon/70-300do_2/

250x500 pixel crops, centre of frame f11
Copy variation

- Left: Addy's 100-400; Right: Frédo's
- (full aperture, 135mm)
Why are lenses so complex?

• It’s not so easy to send light where it should go

source: canon red book

Tuesday, April 28, 2009
Simple lenses are not so good

Plate 11.2 Aberrated imagery from a simple biconvex lens
The image of simple regular patterned subject shows increasingly poor quality off axis and the two uncoated surfaces of the lens both reflect the light source.
Complex lenses are better!

Plate 11.1 Imaging by simple and compound lenses
(a) Simple biconvex one element lens of focal length 100 mm and diameter 50 mm giving $f/2$. Note poor edge detail and low overall contrast. (b) Same lens stopped down to $f/11$. Quality and contrast have improved. (c) A well-corrected five-element 105 mm lens used at $f/11$. 

From Ray's Applied Photographic Optics
Question?
Lens 101 review
Thin lens optics

- Simplification of geometrical optics: All parallel rays converge to one point on a plane located at the focal length $f$.

- All rays going through the center are not deviated.
  - Hence same perspective as pinhole.

TODAY WE GO BEYOND THESE SIMPLIFICATIONS
View #1 of lenses: Geometrical

• Snell’s law bends geometrical rays
  – I mean, Descartes’ law

• Most aberrations can be expressed in this framework
View #2 of lenses (Wave/Fermat)

- Light is focused because all paths have same length
  - Higher index of refraction (speed of light) compensates for length
  - Constructive interference
Consequences on image quality

- Geometrical optics: hard to focus all rays
- Wave optics: diffraction problems
Geometrical perspective
• Snell’s law bends geometrical rays
Thin lens optics

- Simplification of geometrical optics for well-behaved lenses
- All parallel rays converge to one point on a plane located at the focal length $f$
- All rays going through the center are not deviated
  - Hence same perspective as pinhole
Simplification of first-order optics

- Snell’s law: $\eta_1 \sin \theta_1 = \eta_2 \sin \theta_2$
- First order/thin lens optics: use $\sin \theta = \theta$
Third-order optics

- $\sin \theta = \theta - \theta^3/6$
- The extra term leads to third-order aberrations
Third-order aberrations
Spherical aberration

- Rays don’t focus at same position

source: Hecht Optics
Comatic aberration

This is the phenomenon where the diagonal light rays do not focus on one point on the image surface.

This is the phenomenon where there is a tail like that of a comet.

- Inward coma
- Outward coma

Off-axis parallel pencil of rays

Optical axis

source: canon red book
Comatic aberration
Astigmatism

Figure-21 Astigmatism

This is the phenomenon where there is no point image

Principle ray

Lens

Optical axis

Sagittal image

Meridional image

source: canon red book
Defects

Photo-2 The photographs are magnifications of the subject and surrounding area from part of a test chart photographed with a 24mm x 36mm film frame and printed on quarter size paper.

Almost ideal image formation

Peripheral part magnified

① Example of spherical aberration

①-1 Example of inward coma

③ Example of astigmatism

②-2 Example of outward coma

source: canon red book
Curvature of field

Figure-22  Curvature of field

This is the phenomenon where a good image focus surface is bent.

- This is an ideal lens with no image bending.

Subject surface  Focus surface

Subject  Occurrence of image bending

Photo-5  Example of curvature of field

Focusing on center of screen causes corners to go out of focus.

Photo-6  Example of curvature of field

Focusing on corners of screen causes center to go out of focus.

source: canon red book
Curvilinear distortion

Figure 6.10 The effects of curvilinear distortion. (a) The selection of a geometrically incorrect ray bundle by asymmetric location of the aperture stop. (b) Image shape changes caused by barrel and pincushion distortion.
Radial distortion

source digital outback

Fix with PTLens 4.1
Why spherical lenses?

• Because they are easy to manufacture
• (Start from whatever shape, if you grind enough, it will become spherical)

From Optical System Design by Fisher and Tadic
Aspherical lenses

Spherical aberration of spherical lens

Focal point alignment with aspherical lens

source: canon red book
Aspherical lenses

- Harder to manufacture ➔ used with parsimony

source: canon red book
Chromatic aberrations
Chromatic aberration

• The previous aberrations depend on wavelength (because of varying index of refraction)

source: canon red book
Achromatic doublet

Figure 6.38 An achromatic doublet. The paths of the rays are much exaggerated.
Apochromatic & others

- Optimize for multiple wavelengths

Figure 5. Principles of color correction. The colored faces are known as the secondary spectrum.
Fluorite

- Low dispersion

source: canon red book
Bottomline

- Fighting chromatic aberrations is the main cause of lens complexity
Diffraction
(a) The shadow of Mary's hand holding a dime, cast directly on 4 x 5 Polaroid A.S.A. 3000 film using a He-Ne beam and no lenses. (Photo by E. H.)
(b) Fresnel diffraction of electrons by zinc oxide crystals. (After H. Boersch from Handbuch der Physik, edited by S. Flügge, Springer-Verlag, Heidelberg.)
(c) Diffraction through an aperture with varying λ as seen in a ripple tank.

From Hecht's Optics
Airy patterns

- Image of an infinitesimal point
  - for disk aperture
- Absolute limit on lens resolution
- Important at small aperture
- Critical for microscope & telescope

Formally:
- Bessel function
  - Fourier transform of disk
  - Equivalent of sinc for the box

From Hecht's Optics
Diffraction & Fourier

- Aperture Fourier transform
Fraunhofer diffraction

- Incoming coherent plane wave
  - e.g. light coming from a star
  - should image into perfectly sharp point
Fermat principle

- Converge at focal length $\iff$ same path duration (aka optical length)
  - peripheral paths spend more time in air
  - but less in lens (lens is thinner at periphery)
  - light is slower in glass
Refraction and the Lifeguard Problem

- Running is faster than swimming
Fermat principle

- Converge at focal length $\iff$ same path duration (aka optical length)
- Wave contributions remain coherent $\implies$ interfere positively
Fermat principle

- Converge at focal length $\iff$ same path duration (aka optical length)
- Wave contributions remain coherent $\implies$ interfere positively
- Same for any direction, just different point

Diagram:
- Lens
- Mono-chromatic plane wave
- Same optical path length in blue
- Focal plane
Huygens-Fresnel principle

- Each point of the aperture is equivalent to a source of spherical “wavelets”
  - kind of superposition principle
  - All “synchronized” (coherent)
- When no obstacle, their superposition is still a plane wave
Diffraction with lens

- Contribution at a point on sensor
  - same Fermat minimal paths as before
  - Contribution: integral over paths
Integral at a point on sensor

- Wavelets lower in aperture have longer path
  - \( \text{path}(A,P) = \text{path}(B',P) \)
  - \( \text{phase}(A) = \text{phase}(B) \)
  - \( ||BB'|| = ||AB|| \sin \theta \)
    (where \( \theta \) is deviation from optical axis)
  - \( dt = ||BB'||/c \)
    where \( c \) is the speed of light
Integral at a point on sensor

- point at $z$ from $A$:
- $dt(z) = z \sin \theta / c = kz$
Integral at a point on sensor

- point at z from A:
- $dt(z) = z \sin \theta / c = kz$
- wave: $x(A,t) = x(z,t) = X(t) = e^{i\omega t}$
- $x(z',t) = x(z, t - dt(z)) = X(t) e^{-i\omega dt}$
Integral at a point on sensor

- \( x(z',t) = x(z, t - dt(z)) = X(t) \ e^{-i\omega dt} \)

- contribution at \( P \):
  \[
  \int_{z} X(t - \text{path}(A, P)) \ e^{-i\omega k z}
  \]
Integral at a point on sensor

- average power contribution at P: $\left| \int_z e^{-i\omega k z} \right|^2$
- Looks a lot like Fourier power spectrum for frequency $k \omega$
- Note that $k$ increases when P moves up
  - (linearly if the sine is linearized)
Summary

- Diffraction => power spectrum
- Huygens Fresnel: point waves at aperture
- Integrate those little waves, offset by path length => integrate sine waves
  => Fourier

focal plane

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Effect of aperture size

• When we scale down a function
• Scale up power spectrum
• smaller aperture => More diffraction
Lens diffraction

Lens diffraction

- See also [http://www.cambridgeincolour.com/tutorials/diffraction-photography.htm](http://www.cambridgeincolour.com/tutorials/diffraction-photography.htm)
The idiocy of Megapixels

Questions?
Other quality issues
Flare

Figure 5.6  Formation of flare spots by a simple lens. Images of the source are formed at distances $A$ and $B$, where:

\[ A = \left( \frac{n - 1}{an - 1} \right) f \quad B = \left( \frac{n - 1}{bn - 1} \right) f \]

and $a = 2, 4, 6 \ldots$, $b = 3, 5, 7, \ldots$ For $n = 1.5$, $A = f/4, f/10, f/16$ etc. and $B = f/7, f/13, f/19$ etc.

From "The Manual of Photography" Jacobson et al
Example of flare "bug"

- Some of the first copies of the Canon 24-105 L had big flare problems
Use a hood! (and a good one)

Flare ray

**Hood is to short**

Flare

Good hood

Adapted from Ray’s Applied Photographic Optics
Plate 15.1  Lens flare with an uncoated lens
(a) Flare effects. (b) Reduction of flare by use of a lens-hood.
Coating

• Use destructive interferences
• Optimized for one wavelength

Figure 5.7  An anti-reflection coating on glass using the principle of destructive interference of light between reflections $R_1$ and $R_2$

Figure 5.8  The effects on surface reflection of various types of anti-reflection coatings as compared with uncoated glass (for a single lens surface at normal incidence)
Vignetting

- http://www.photozone.de/3Technology/lenstec3.htm
Vignetting

• The periphery does not get as much light

source: canon red book
Quality evaluation
LPIs

• Line pair per inch

Input

After lens

http://www.imatest.com/docs/sharpness.html

http://www.imatest.com/docs/tour.html
Sharpness

$\delta_0 = 1 \times 10^4$ lp/mm; $\text{flens} = 61$ lp/mm; $\text{lord} = 2$

Sine pattern: Original
Sine pattern: Lens only
Bar pattern: Original
Bar pattern: Lens only

Amplitude

MTF %

Line pairs per mm; MTF = 50%, 10% @ 61, 183/mm

Tuesday, April 28, 2009
MTF

- Modulation Transfer Function
- Pretty much Fourier transform of lens response
- Complex because needs to be measured at multiple location

Table 3

<table>
<thead>
<tr>
<th>Spatial frequency</th>
<th>Maximum aperture</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>S</td>
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<tr>
<td>10 lines/mm</td>
<td></td>
</tr>
<tr>
<td>30 lines/mm</td>
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</tbody>
</table>

Graph 5 MTF Characteristics

Here the x axis is image location

source: canon red book
Blur index based on Photoshop!

- Lens sharpness (or lack thereof) expressed as the amount of Photoshop blur that would blur the image similarly

- 50mm f/1.4 [http://www.slrgear.com/reviews/showproduct.php/product/140/sort/2/cat/10/page/2](http://www.slrgear.com/reviews/showproduct.php/product/140/sort/2/cat/10/page/2)
Lens design
Optimization software

• Has revolutionized lens design
• E.g. zooms are good now

Figure 11.50 An example of the kind of lens design information available via computer techniques. (Photos courtesy Optical Research Associates.)
Lens design, ray tracing

source: canon red book
Optimization

• Free parameters
  – Lens curvature, width, position, type of glass
  – Some can be fixed, other vary with focal length, focus (e.g. floating elements)
  – Multiplied by number of lens elements
• Energy/merit function
  – MTF, etc.
  – Black art of massaging the merit function
• Optimize for
  – All image locations
  – All wavelengths
  – All apertures
  – All focusing distances
  – All focal lengths (zoom only)
• Usually uses simulated annealing
References
Links

- http://www.dpreview.com/learn/?/key=chromatic+aberration
- http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/aberrcon.html#c1