LECTURE 15

• **Readings:** Finish Section 6.2

Lecture outline

• Big Screw
• Quiz 2
• Review of Poisson process
• Merging and splitting
• Random incidence
Big Screw

- Alpha Phi Omega Institute Screw Contest
- Donations are anonymous

Quiz 2

- April 11, during lecture
- Coverage is through Bernoulli processes
Review: Poisson process with rate $\lambda$

- Defining characteristics:
  - **Time homogeneity**: $P(k, \tau)$
  - **Independence**
  - **Small interval probabilities**: For very small $\delta$,

$$P(k, \delta) \approx \begin{cases} 
1 - \lambda \delta, & \text{if } k = 0; \\
\lambda \delta, & \text{if } k = 1; \\
0, & \text{if } k > 1.
\end{cases}$$

- $N_\tau$ is a Poisson random variable with parameter $\lambda \tau$:

$$P(k, \tau) = \frac{(\lambda \tau)^k e^{-\lambda \tau}}{k!}, \quad k = 0, 1, \ldots$$

- Independent exponential interarrival times
- Time of $k$th arrival: Erlang of order $k$
Adding (merging) Poisson processes

- Merging indep. Poisson processes gives a Poisson process

\[ f_{Y_k}(y) = \lambda_k y^{k-1} e^{-\lambda y} (k-1)! , \quad y \geq 0 \]

- First-order interarrival times \((k = 1)\):
  \[ f_{Y_1}(y) = \lambda e^{-\lambda y} , \quad y \geq 0 \]

- By-product: Sum of independent Poisson random variables is a Poisson random variable
Example: Three similar light bulbs

- Three light bulbs have independent lifetimes
  \[ X_1, X_2, X_3 : \text{ exponential with parameter } \lambda \]

- Install all three. Let \( T \) be the time when the last of the three burns out. Find \( E[T] \) and \( \text{var}(T) \).
Merging Poisson processes (again)

- Merging indep. Poisson processes gives a Poisson process

\[
f_{Y_k}(y) = \lambda_k y^{k-1} e^{-\lambda y} (k-1)!
\]

- What is the probability that the next arrival comes from the first process?
Example: Two dissimilar light bulbs

- Two light bulbs have independent lifetimes
  
  \[ X_1 : \text{exponential with parameter } \lambda_1 \]
  
  \[ X_2 : \text{exponential with parameter } \lambda_2 \]

- Install both. Let \( T \) be the time when the last burns out. Find \( E[T] \).
Splitting of a Poisson process

- Suppose email traffic through server is a Poisson process and destinations are independent

- Each output stream is Poisson. Why?
Example: Geometric number of exponential RVs

- Let $Y = X_1 + X_2 + \cdots X_N$ where
  
  each $X_i$ : exponential with parameter $\lambda$
  
  $N$ : geometric with parameter $p$

  and $N, X_1, X_2, \ldots$ are independent

- What is the distribution of $Y$?
Random incidence in Poisson process

- Poisson process that has been running forever
- Show up at some “random time” ("arbitrary")

What is the distribution of the length of the chosen inter-arrival interval?
Random incidence in Bernoulli process

• You watch a player shoot free throws on several days. On average, the length of the streak of made free throws when you walk into the gym is 10. Assuming free throws are independent, what is his probability of making any one free throw?
More random incidence

- 4 buses carrying 148 job-seeking MIT students arrive at a job convention. The buses carry 40, 33, 25, and 50 students, respectively.

- One of the students is selected randomly . . .

- One of the buses is selected randomly . . .