The Visibility Problem
and
Binary Space Partition

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Slides mostly by Nati Srebro
The Visibility Problem
Painter’s algorithm

- Draw objects in order, from back to front
- Can one always order objects from front to back? That is, is “A occludes B” a partial order?
- Assuming:
  - Simple objects, e.g., segments or triangles
  - Objects disjoint
- In 2D: No
- In 3D: No
- We will have to split sometimes
Binary Planar Partitions
Painter’s Algorithm
Binary Planar Partitions
Auto-partitions
Auto-partitions
What is the complexity of BSP using auto-partitions?

\[1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} = O(n^2)\]
Binary Planar Partitions

Goal:
Find binary planer partition, with small number of fragmentations
Random Auto-Partitions

Choose random permutation of segments

\((s_1, s_2, s_3, \ldots, s_n)\)

While there is a region containing more than one segment,
separate it using first \(s_i\) in the region
Analysis

$u$ can cut $v_4$ only if $u$ appears before $v_1, v_2, v_3, v_4$ in random permutation

$P(u \text{ cuts } v_4) \leq 1/5$
Random Auto-Partitions

\[ E[\text{number of cuts } u \text{ makes}] = E[\text{num cuts on right}] + E[\text{num cuts on left}] \]
\[ = E[C_{v_1} + C_{v_2} + \cdots] + E[C_{t_1} + C_{t_2} + \cdots] \]
\[ = E[C_{v_1}] + E[C_{v_2}] + \cdots + E[C_{t_1}] + E[C_{t_2}] + \cdots \]
\[ \leq 1/2 + 1/3 + 1/4 + \cdots + 1/n + 1/2 + 1/3 + 1/4 + \cdots + 1/n \]
\[ = O(\log n) \]

\[ E[\text{total number of fragments}] = n + E[\text{total number of cuts}] \]
\[ = n + \sum_u E[\text{num cuts } u \text{ makes}] = n + nO(\log n) = O(n \log n) \]
Random Auto-Partitions

Choose random permutation of segments

\((s_1, s_2, s_3, \ldots, s_n)\)

While there is a region containing more than one segment,
separate it using first \(s_i\) in the region

\(O(n \log n)\) fragments in expectation
What about 3D?

• Assume arbitrary order of triangles
• What is the complexity of the BSP?
  – Each triangle can be split using $n-1$ planes
  – From the perspective of the triangle, it is split using $n-1$ lines
  – Complexity of arrangement: $O(n^2)$
  – Total complexity: $O(n^3)$
• Can be improved to $O(n^2)$
What about 3D ctd

- Also, at least $\Omega(n^2)$
  - Auto-partitions
  - General
Algorithms

• Z-buffer:
  – Draw objects in arbitrary order
  – For each pixel, maintain distance to the eye ("z")
  – Only draw pixel if new "z" is closer

• Painter’s algorithm:
  – draw objects in order, from back to front